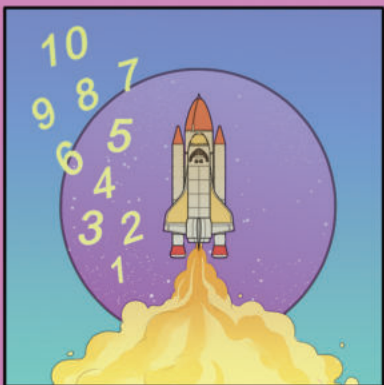
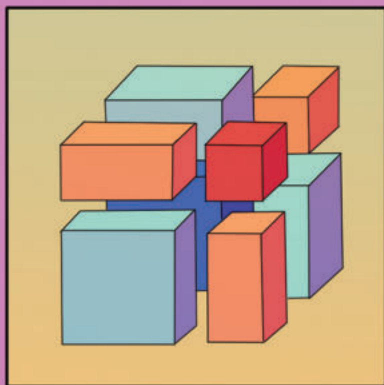
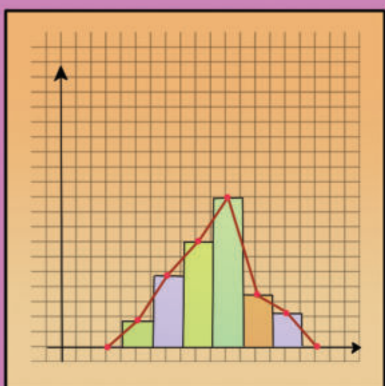
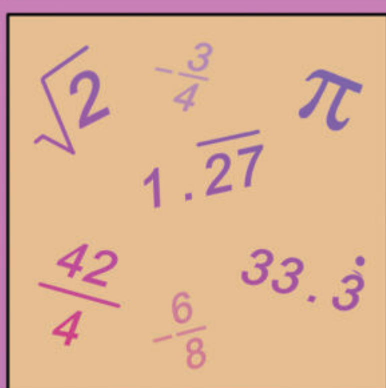
# Mathematics

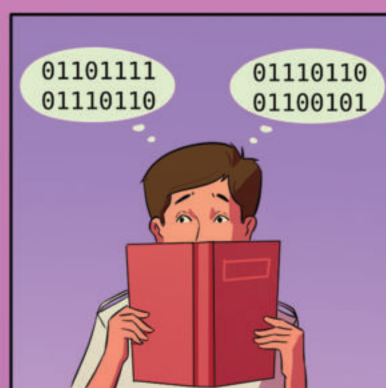
## Class Eight










NATIONAL CURRICULUM AND TEXTBOOK BOARD, BANGLADESH


বিনামূল্যে পাঠ্যপুস্তক বিতরণ

২০১০ সাল থেকে শেখ হাসিনা সরকার প্রাথমিক স্তর থেকে মাধ্যমিক স্তর পর্যন্ত বিনামূল্যে পাঠ্যপুস্তক শিক্ষার্থীদের মধ্যে বিতরণ করে আসছে। প্রতি বছর ডিসেম্বরের শেষ সপ্তাহে মাননীয় প্রধানমন্ত্রী শেখ হাসিনা বিনামূল্যে পাঠ্যপুস্তক বিতরণ কার্যক্রম শুভ উদ্বোধন করেন। তারই ধারাবাহিকতায় জানুয়ারির ১ তারিখেই শিক্ষার্থীরা উৎসবমুখর পরিবেশে পাঠ্যপুস্তক হাতে পায়। ফলে শিক্ষার্থী ঝরে পড়ার হার কমেছে এবং বিদ্যালয়ে শিক্ষার্থী অন্তর্ভুক্তি দিন দিন বেড়েই চলেছে। জানুয়ারির ১ তারিখ এখন পরিণত হয়েছে পাঠ্যপুস্তক উৎসবে। ২০১০ থেকে ২০২৩ শিক্ষাবর্ষ পর্যন্ত মোট ৪৩৪ কোটি ৩ লক্ষ ৪৬ হাজার ৩৬৬টি বই বিনামূল্যে শিক্ষার্থীদের মাঝে বিতরণ করা হয়েছে।

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# Mathematics

## Class VIII

(Experimental Edition)


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# PREFACE

In this ever-changing world, the concept of life and livelihood is changing every moment. This process of change has been accelerated due to the advancement of technology. There is no alternative to adapting to this fast changing world as technology is changing rapidly ever than before. In the era of fourth industrial revolution, the advancement of artificial intelligence has brought about drastic changes in our employment and lifestyles that will make the relationship among people more and more intimate. Various employment opportunities will be created in near future which we cannot even predict at this moment. We need to take preparation right now so that we can adapt ourselves to that coming future.

Although a huge economic development has taken place throughout the world, problems like climate change, air pollution, migrations and ethnic violence have become much more intense nowadays. The breakouts of pandemics like COVID 19 have crippled the normal lifestyle and economic growth of the world. Thus, different challenges as well as opportunities, have been added to our daily life.

Standing amid the array of challenges and potentials, sustainable and effective solutions are required to transform our large population into a resource. It entails global citizens with knowledge, skill, values, vision, positive attitude, sensitivity, adaptability, humanism and patriotism. Amidst all these, Bangladesh has graduated into a developing nation from the underdeveloped periphery and is continuously trying to achieve the desired goals in order to become a developed country by 2041. Education is one of the most crucial instruments to attain the goals. Hence, there is no alternative to the transformation of our education system. This transformation calls for developing an effective and updated curriculum.

Developing and updating the curriculum is a routine and important activity of National Curriculum and Textbook Board. The curriculum was last revised in 2012. Since then, more than a decade has elapsed. Therefore, there was a need for curriculum revision and development. With this view, various research and technical studies were conducted under NCTB from 2017 to 2019 to analyze the current state of education and identify the learning needs. Based on the researches and technical studies, a competency-based and seamless curriculum from K−12 has been developed to create a competent generation capable of surviving in the new world situation.

Under the framework of this competency based curriculum, the textbooks have been prepared for all streams (General, Madrasah and Vocational) of learners for Class Eight. The authentic experience-driven contents of this textbook were developed with a view to making learning comprehensible and enjoyable. This will connect the textbooks with various life related phenomenon and events that are constantly taking place around us. It is expected that, through this, learning will be much more insightful and lifelong.

In developing the textbooks, due importance has been given to all − irrespective of gender, ethnicity, religion and caste while the needs of the disadvantaged and special children are taken into special considerations.

I would like to thank all who have put their best efforts in writing, editing, revising, illustrating and publishing the textbook.

If any errors or inconsistencies in this experimental version are found or if there is any suggestions for further improvement of this textbook, you are requested to let us know.

**Professor Md. Farhadul Islam**
Chairman
National Curriculum and Textbook Board, Bangladesh

## Dear students,

As you know, National Curriculum and Textbook Board, Bangladesh has prepared textbooks according to curriculum for all secondary level students. The new textbooks also bring radical changes to the way you learn Mathematics. The things that have been emphasized while developing this book are - to create opportunities for students to solve mathematical problems through hands-on work by observing the objects and events of the familiar environment around them and to show the way to use mathematical skills in everyday life. The teacher will help you in this journey to discover the joyful world of mathematics.

A total of ten learning experiences are planned for you in the class VIII book. You will participate in these experiences mathematically analyzing and solving real-life problems. Each learning experience is presented step by step so that you can master mathematical concepts and skills through active participation and use of real materials. This journey of learning mathematics through mathematical inquiry will be enjoyable for you as you will discover for yourself the relation of mathematical concepts to real life. In this learning process the textbook will serve as a helpful tool for you.

Your teacher will provide full support in all activities inside and outside the classroom. We also hope that you will be supportive of each other as you participate in the various activities of this learning program and enjoy the joy of mathematics with your classmates. You will always remember that we can accomplish anything successfully when we all have a cooperative spirit. We hope, this book will play an important role in ensuring an effective and enjoyable learning journey for you in the world of mathematics.

Good luck to you all.

# Index

# Mathematical Exploration

## You can learn from this experience

- Process of Mathematical Exploration
- Steps of Mathematical Exploration
- Pattern
- Method of Verifying the Reliability of Sources of Information

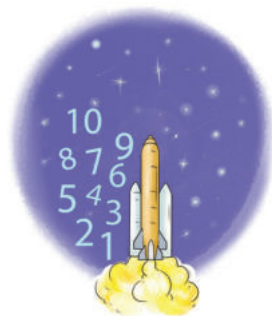
# Mathematical Exploration

In your previous classes you got some interesting information about numbers and their properties. Today we will not study any new property of numbers. On the other hand, we will learn how to explore about the properties of numbers. Do you remember collecting information on various topics and analyzing them in your previous class? Similarly, mathematicians and researchers collect and explore several facts about numbers around us. What do you think about that? Is it a fun job or a boring one investigating like them? Let's explore a problem like mathematical researchers and decide ourselves.

The problem we are working on today is one of the favorite problems of renowned mathematicians around the world and it is related to sequence of cardinal numbers. You can find some relation on your own after completing the exploration. In addition, you may discover something which nobody found before. While doing the exploration, you'll do some calculations and note-taking. In the end you'll be amazed thinking about the journey you have made, and the things you learn. So, let's start the problem step by step.


In the beginning let's review what are cardinal numbers. Did you notice that numbers are coming one after another? Like 1, 2, 3, … etc.?

Again, did you notice while launching a rocket or spaceship, numbers are counted backwards like 10,9,8,7,…? This is called count-down.




"Let's fly in the sky"

Again, we have examples of increasing sequence of numbers. Notice that the page number of the book increases by one when we keep turning each page, doesn't it?


We are calling this sequence of cardinal numbers. This is nothing complex, just placing integers one after another.

Let’s go to the main problem. Are you ready with pen and paper?

1 The problem starts with numbers. Take any four consecutive integers and write them by their order. Keep some space between any two numbers so that you can write something. As an example, we have taken four numbers here. You can take any other four numbers of your choice.

4 5 6 7


2 Now your job is to put addition (+) or subtraction (-) signs between the numbers. Without changing the order of numbers put + or – signs in various ways. Somewhat like this:

$4 + 5 - 6 + 7$

$4 - 5 + 6 + 7$

3

## Guess the answer!

- In how many ways you can put addition or subtraction signs between the numbers?
- Write your answer in the empty box.

Now put + or – in all possible ways.

Did you try every combination in you thought of?

Did you try using all signs as + and all signs as - ?


4 The next step is counting the results of all combinations. Do the necessary addition and subtraction and write down the results. For example

4 – 5 + 6 + 7 = 12

5

## Now guess the following answers

- Among the results, what can be the highest value and what can be the lowest?
- If we order them from smaller to greater values, can we find any relation?

6 Do you observe any special sequence or relation among the results obtained?

If you notice any interesting sequence or relation, write them down in your notebook. To help your thought process, perform the tests given below.

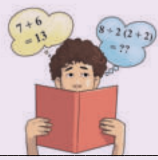
7

## Test on your own

[To sharpen your thinking four questions are given here. If more questions arise in your head, write them below or in the next page.]

1. If we arrange the results in ordering, will their differences remain the same?
2. Can any result be zero?
3. Can we get repeated results?
4. Can any result be greater or smaller depending on + or – signs?
5. 
6. 

**8** In the box below, write down six of the most interesting properties you have found out about the results. Also write down your reasons behind those properties.




Did you make any errors while performing the test? Did you have to recount any result? Did you have to go back to previous step and repeat any time? If you faced these, write them down in the box below.

## Individual Task

We are giving some more questions for your practice. Finding answers to this type of questions will help you improve your intelligence. Perform the following tests in the same way you did the above ones -

- If we order the numbers greater to smaller which is opposite to the first exploration, what are the results?
- Take three consecutive numbers instead of four.
- Work with more than four consecutive numbers.
- Take four different numbers. Do you notice any similarities between the results of the first exploration and this one? Are there any differences?

## Process of Mathematical Exploration

Did you like the above exploration involving numbers? This was just one example of number investigating problem. In mathematics, there are many other branches except investigating numbers, you can have many questions and problems regarding them. They are related to our daily life. The process of finding the properties, solutions or answers to a problem is called mathematical exploration. Apart from mathematics, exploration is done in other subjects too. For example, scientific explorations are done in laboratories. Again, the exploration of social sciences is done in various social institutions. The most interesting thing about mathematical exploration is that you can do it at your home using only pen and paper. Sometimes you may need calculator or computers. But is there anything more interesting than doing these explorations by hand calculations?

## Steps of Mathematical Explorations

As we have mentioned before, we will not study any new topic of mathematics in this experience. We will just learn the process of mathematical explorations. Already you have completed one exploration on your own. We hope you had fun doing that. Now let’s look back at the process of the work, which steps we followed. The steps we followed are given below, but not according to order (A-I). Your task here is to write the order on the blank circle at left. IF any step was not needed during our exploration, you may keep the corresponding left circle blank.

A. Estimated the probable result.

B. Analyzed the problem and understood it.

C. Matched the estimated result with the result obtained from the experiment.

D. Checked if our process is alright.

E. Identified the question/problem.

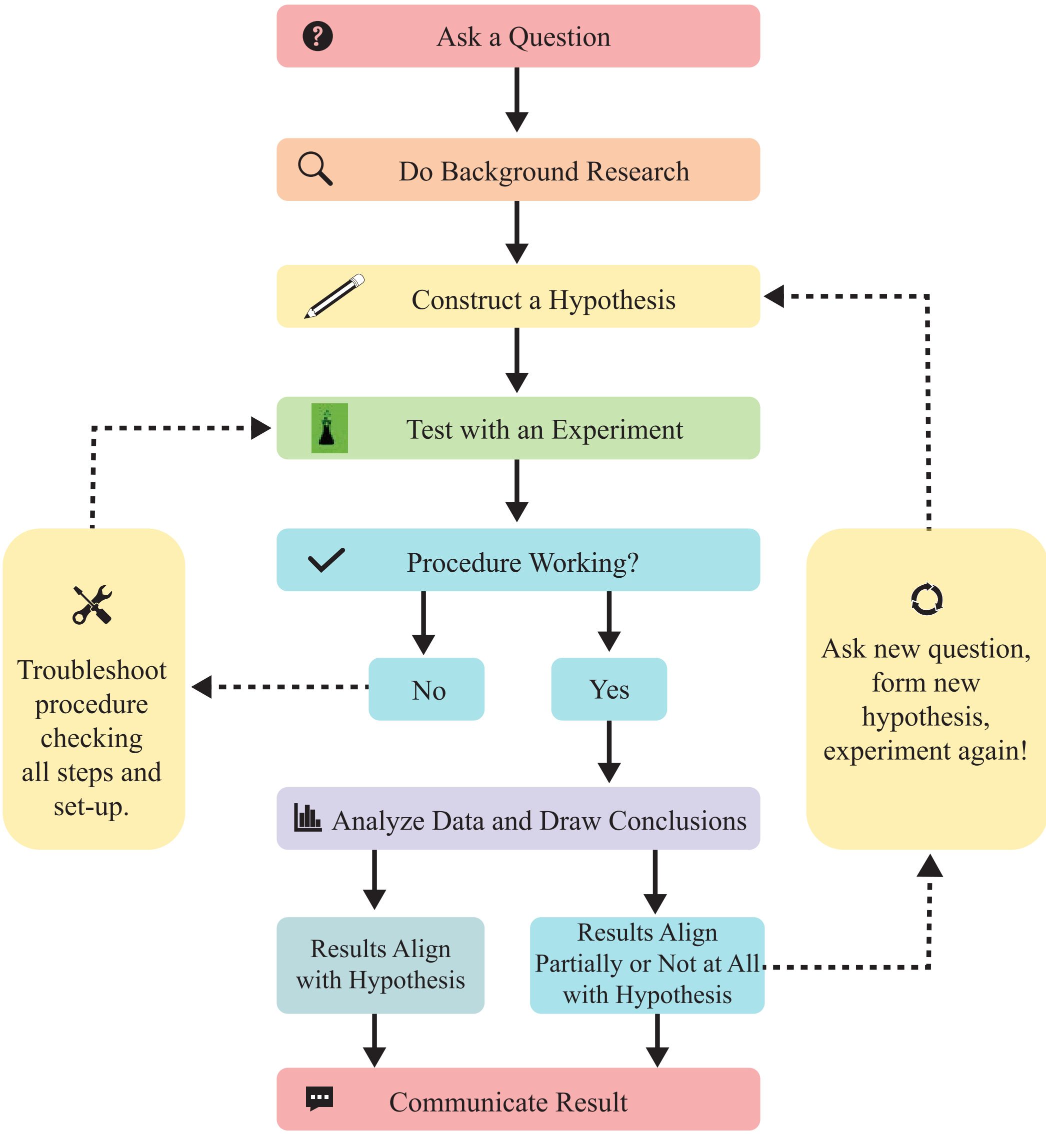
F. Analyzed the results.

G. Repeated the previous step if there was any error.

H. Noted down our observations.

I. Tested our estimations.

Now notice, if you had done the tasks without any order, would you get the same result? Would you get the answers? Of course, not. It means there is a specific order of task while performing a mathematical exploration. All explorations will not follow all the same steps, but there should be a general order of these tasks. The process followed by mathematicians and scientists in their various research areas can be expressed using a common diagram. We are giving an example of that type of diagram. You can also call it a flow-chart for mathematical exploration. Pair up with one of your classmates and find similarities or differences of the steps mentioned below with the steps of your exploration. Here, our estimated result is also known as hypothesis.



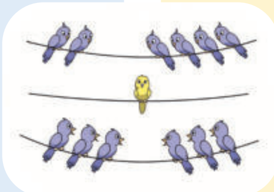

Flow chart for mathematical exploration process

## Group Work

Surely there are some similarities and differences between the flow chart given in last page and the flow chart of the exploration you did. Discuss with your partner and note down the similarities and differences you two find out:

**Similarities**

**Differences**



Notice that various steps of the above flow chart are given in the table below. You found some similarities between the steps of your exploration and steps given in the flow chart. Note down the specific work you did at a particular step in the table below. One example is done for you:

| Step | Work |
|---|---|
| Identifying the problem | What are the properties of the results if we put addition or subtraction in between four consecutive numbers? |
| Estimating the result | |
| Testing | |
| Identification of errors | |
| Analyzing the result | |

## Why is mathematical exploration important?

Up to today we became familiar with the steps of mathematical exploration. We also explored one problem on our own. Are you curious about the outcomes of mathematical explorations? Why do the mathematicians and scientists explore various topics? Obviously, they want answers to unknown problems, but what is the benefit of that knowledge? Write down your own thoughts about this in the blank space below. If you want, you can draw some picture too.

There could be many reasons why we perform mathematical exploration. The main reason among them is to understand the properties of a problem at a deeper context. All problems always cannot be solved instantly. But if we understand the problem better than the last time, we proceed closer to solving the problem way more than before.

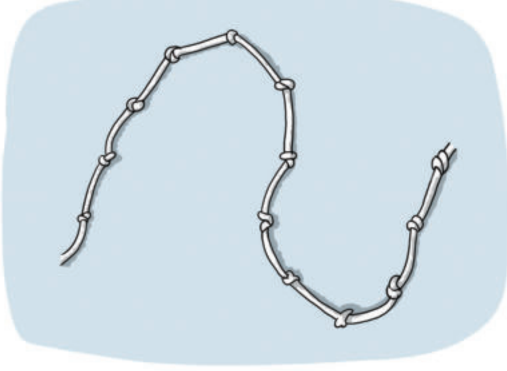
Again, understanding the properties of a problem also helps us solve similar kinds of problems. In your previous classes you have learnt to use various algebraic formulae. We can solve many similar algebraic problems using the similar formula, can't we?

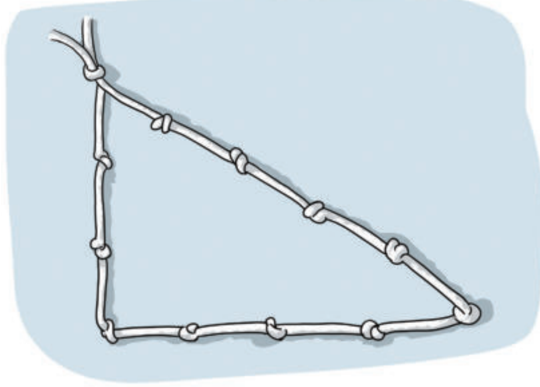
## Teamwise exploration

So far you have come to know what mathematical exploration is, how to complete this exploration, what are its steps. To solve various maths problems in the same way in future, we need to practice regularly from now on. So, we will do some teamwork this time. The teacher will divide your whole class into six groups. Below are the problems or questions for three of the groups, the remaining three problems will be provided by the teacher. Teams have to explore them and present solutions/observations. There will be a lottery under the guidance of the teacher to determine which problem will be assigned to which group. It is best to use poster paper for presentation, if not, explain your observations according to the Mathematical Inquiry Steps (pages 6-7) as suggested by the teacher. Three problems are given for you. With the teacher's help, you will create two or three more exploration problems and attach them to this experience. So check out the problems or questions –

## Problem 1

The ancient Egyptians made great progress in mathematics and science; you know? They used a rope for various geometrical measurements. The characteristic of the rope was that it had 12 knots at regular intervals (just like the picture on the left side below).





With these ropes they used to make right-angled triangles (as shown in the picture on the right side above). You must know which triangles are right-angled? In a later experience you will learn some interesting uses of right-angled triangles.

Collect a rope with 12 equally spaced knots for your group’s exploration. Then-

1. Find out what kinds of triangles you can make. The condition is that the rope has knots at both ends, one knot at each vertex of the triangle, and the knots at the two ends of the rope meet.
2. What other shapes can you make with such a rope (for example, square, rectangle, etc.)?

## Problem 2

The table below has a total of 196 cells (but better to check the count yourself, we might be wrong). Each row of the table has a number assigned and each column has a English character assigned. From these numbers and letters the address of each cell can be found. For example, the address of the 3rd cell in the 2nd row is 2C.

In this exploration you will basically enter the results of the products of numbers 1 to 10 in the cells indicated in the table below. For example, $2 \times 1 = 2$, write only the product in the specified cell, that is, 2. But the condition is that if there is a digit in the hundred’s place or ten’s place of the product, it can't be placed, only the digit of unit’s place will be placed. For example, $2 \times 5 = 10$, only 0 will be written in the specified cell.

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | | | 2C | | | | | | | | | | | |
| 3 | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | | | |
| 11 | | | | | | | | | | | | | | |
| 12 | | | | | | | | | | | | | | |
| 13 | | | | | | | | | | | | | | |
| 14 | | | | | | | | | | | | | | |

Table-1.1

## Now follow the instructions below:

1. Start writing the products of 1 from cell 3C. That is, 1 will sit in 3C, 2 in 3D, 3 in 3E …. Write the product up to $1 \times 10$ in this way, but remember the condition.
2. In the same way start the products of 2 in 4C, products of 3 in 5C …, continue this and start the products of 10 in 12C.
3. Now put zeros in the cells around the digits to complete the whole table.

## Now observe and answer the following questions:

a. What properties can you notice among the numbers?

b. Do you see any repeats? If you see any repetitions, can you express it/them through cycles? Can repetitions only rotate in the same direction, or also in the opposite direction?

c. Why is the property of the table like this?

## Problem 3

Surely you have an idea about the factors of numbers. Even if you forget, let us remind you – if a number is the product of two numbers, then the two numbers each is a factor of the first number. For example,

Factors of 8 are:

1, 2, 4 and 8,

Now drop 8 and add its factors and see what we get:

$1 + 2 + 4 = 7$

Similarly, write the factors of 10. Now what will be the sum of the factors of 10 excluding 10?

Now let's see what the sum of the factors of 12 is in the same way.

Factors of 12 are:

1, 2, 3, 4, 6 and 12

Apart from 1, 2, 3, 4, 6 and 12, are there any other factors of 12?

Then, if we add the remaining factors after excluding 12, let's see what the sum is-

1 + 2 + 3 + 4 + 6 = 16

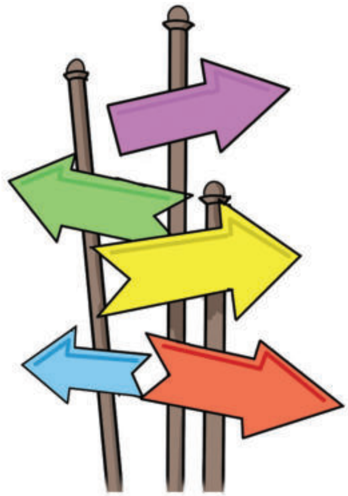
Notice that the sum of the factors of 8 and 10 is less than 8 and 10 respectively. But that is not the case with 12. So, 12 is an abundant number.

## Your exploration task is:

1. Find 5 other abundant numbers.
2. Why are the properties of abundant numbers different from properties of other numbers?
3. Prepare a definition of abundant numbers to explain to classmates in other groups.

## Directions for presenting the exploration

You've already received instructions on how to conduct your team's exploration. Below are some instructions on what to do after the work is done, notice these.



1. Write the answers to the following questions regarding your exploration on required amount of poster papers. Uses figures, tables, or diagrams when necessary.

   a. What is your exploration about?

   b. To find a solution, which questions are required to be answered?

   c. What tools, knowledge and skills did you need to carry out the exploration?

   d. Present the steps in solving the exploration with the help of a flow-chart. If necessary, refer to steps of mathematical exploration of the experience.

   e. What are the results of the exploration?

   f. What can you learn by analyzing the results?

   G. Did you make a mistake in any step? Did you have to go back to a previous step to correct a mistake?

2. Now explain your problem to another group under the guidance of the teacher and exchange your poster or report. See if they understand your exploration. Write down their questions, suggestions and comments. Add their questions, suggestions and comments to your report and refine your report if needed.
3. Check if you understand the other team's poster or report, ask them questions and give suggestions.
4. Neither teachers nor parents will help you directly in this exploration, they will just observe. So, you must figure out the answer to your problem by yourself.
5. If meaningful, demonstrate with real-life objects on the day of the presentation.

## What do we find by mathematical exploration?

You participated in some mathematical exploration as a group and saw the results of other groups' exploration. But have you ever wondered what can be found through mathematical exploration? Below are some possible answers to this question. Put a tick (√) in the box to the left of the answers that match your observations. In addition to the answers below, write anything else that comes to your mind in the blank spaces.

- ☐ Questions are answered.
- ☐ Solution to problems are found.
- ☐ Methods of determining solutions or answers can be discovered.
- ☐ A common solving method for solving similar problems can be fixed.
- ☐ Ability to make decisions on complex issues.
- ☐ New problems can be created.
- ☐ The right method can be found by making errors.

- ☐ New mathematical relationships can be discovered.
- ☐ Solving mathematical problems leads to solutions to problems in other subjects.
- ☐ One's own intellectual development.
- ☐ ____________________
- ☐ ____________________
- ☐ ____________________

[Can you give an example of each of the statements you ticked?]

## Patterns from repetition of properties

Notice in the previous list it is written that similar problems can be solved by mathematical analysis. If you agree with this statement, then we can discuss this a bit. The following two lines contain different expressions of numbers. Observe and say, can you determine the next term of the expressions after the last provided term?

a. 1, 2, 3, 4, 5, 6, 7, ……

b. 1, 2, 4, 8, 16, 32, ……

Which method did you use to determine?

A property of the first expression is: next term = present term + 1;

For the second expression: next term = present term × 2.

So if you can find the properties of any mathematical expression, then you can solve any problem related to that expression. This feature of repetition is called a pattern. One of the tasks of mathematical exploration is to discover repetitions or patterns in the properties of a mathematical problem.

Let's look at another interesting example of pattern discovery from mathematical exploration.

There were many great thinkers before us who explored and discovered the beauty of mathematics. Relying on their knowledge, today we can solve difficult problems in no time. Here is one such person whose picture you can see on the side. He is the famous 12th century Italian mathematician Fibonacci. He is known to have discovered a sequence of numbers by exploring nature. The sequence may seem very simple at first glance. But if you observe carefully, you will see that there is a repetition or pattern of a certain property in it. You need to find out what it is. The sequence is:

Italian Mathematician Fibonacci

**0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ……**

The question for you is, what is the $12^{th}$ number of Fibonacci sequence?

Use this blank space to determine your answer. If you are sure of your answer, you can explain the main property of the sequence and write it down.

## Individual Worksheet

Can you determine any number pattern and its characteristics on your own? Try yorself and submit your findings to the teacher through worksheets.

## Why do we need to look for patterns?

Are you thinking about the requirement of discussing about patterns? So let's try to understand the necessity of pattern through two very common problems of Arithmetic and Algebra. The problems are:

1. What is the 5% of 50?

2. (2 + b)2 =?

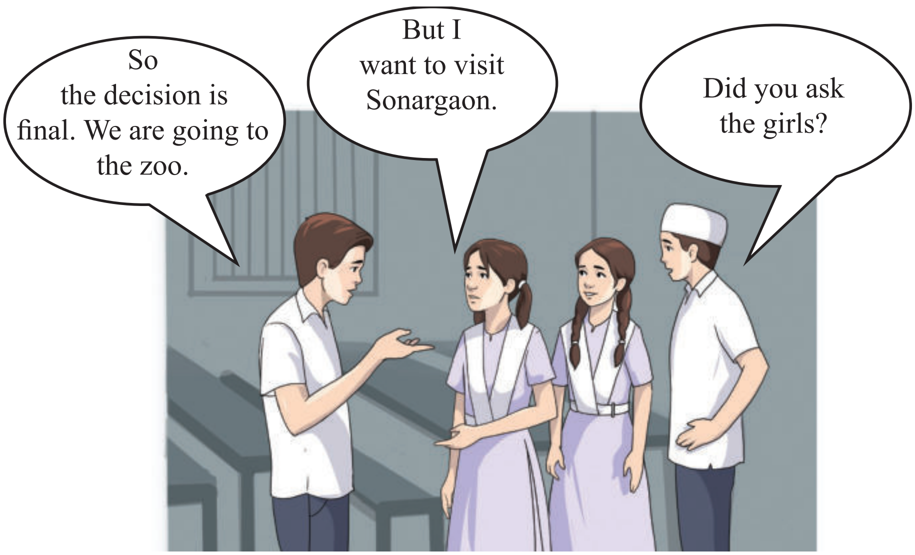
Surely you did not have much trouble to solve the two problems, right? In the first problem, if you had 500 instead of 50 and 25% instead of 5%, your method of solution would have been the same. Again, you would solve the second problem in the same way, if there was a in place of 2 and 29 in place of b.

Thus, it is possible to solve problems of the same nature, or of the same type, using a particular method. So, when doing mathematical exploration we aim to understand the problem pattern. The method of solving a particular type of problems is called a formula. If we cannot discover patterns, we cannot find solutions to problems. In the later experiences of this book, you will find many problems that will be interesting to think about. If you look at the problems as individual mathematical explorations, solving them becomes much easier once you understand their patterns.

## Sources of information in mathematical exploration

In this experience we discussed steps in mathematical exploration and observation of patterns through exploration. But in real life every mathematical exploration involves collecting data and information. That information or data is collected from various prescribed sources. The reliability of the sources of information is as important as information or data for mathematical exploration. An example will make it easier to understand.

If I tell you a story, you will understand easily. Think you are a student of the class eight 'Joba' section; You are a total of 45 children. All classes in your school are planning to go on a study trip. But you cannot decide where to go. Your friend Anik came to know from the class eight 'Surjamukhi' branch that they visited the zoo. After hearing from Anik, the boys of your class decided that the students of class eight 'Jaba' branch should also go to the zoo.




Thinkfor a while and tell if there is any mistake in the process of this decision making. If there is any, write them down.

So, can you understand how important it is to collect information or data from reliable sources for making correct/effective decisions? Can you explain in your own words why it is important to use reliable sources to collect information or data?

This was about data collection in a limited scale. Let's look at the importance of reliability of data sources on a larger scale.

You must remember that in the year 2020, the covid-19 pandemic started a terrible disaster all over the world. During that difficult time, scientists from different countries worked tirelessly and successfully discovered vaccines, saving people from all over the world from death. But do you know how scientists discovered these vaccines? How did they test if these vaccines are effective for humans?

The formulation of any vaccine is the result of a scientific research carried out in several steps. An important step in this is to collect information and data and analyze it to arrive at a conclusion. Researchers collect data and information by using them different numbers of people and other animals in several stages. Then we get the vaccines when they are analyzed. In the first experience you have learned about the stages of data analysis in any research.

Finding a reliable source to gather this information is one of the most important tasks. Now imagine that scientists vaccinate 100 people in your area, and say that they will come back next month to collect the information on how the vaccinated people are doing. But in the next month, 65 of those 100 people moved to other areas. Now if scientists collected data from any 100 people in your area and developed a vaccine based on that, would you take that vaccine?

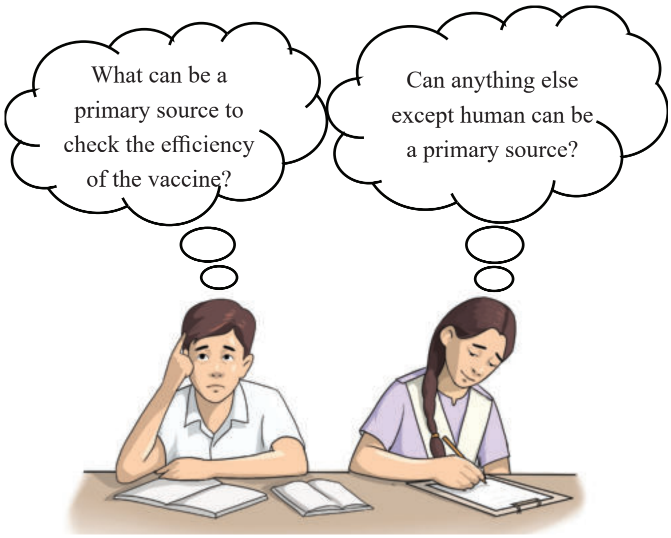
☐ Yes ☐ No

Why?

That means, if the research or exploration data is not collected from reliable sources, its results are of no use to us, right? Because if the source is not selected properly then the information and data will not be correct, and the necessary conclusions cannot be reached. Even in the case of vaccine development, if scientists could not select the right sources to collect data, it would not be possible for us to have an effective vaccine to prevent the Covid-19 pandemic.

**Checking the reliability of the source**

Sources of information are of two types: a) human sources and b) non-human sources.




A human source is when a human is giving you information directly/indirectly. On the other hand, when you have to observe an something actively to collect data is called an non-human source. Again, it is said that the data collected directly from classmates or family for research are primary data. And the data that you have collected indirectly from school records or any other reliable source are secondary data. We can also say that the source from which we get primary data are primary sources. Sources that record information obtained from primary sources such as papers, reports or documents are secondary sources of information.

**Properties of a reliable source of information are:**

- ☐ Credibility of the source with the topic.
- ☐ Acceptance of the source depending on the required information.
- ☐ Trustworthiness of the source.
- ☐ Representativeness of the source.

Knowing about the different types of sources, can you say which type of data is more reliable? Surely primary data, right? Reason – Primary data is collected directly from the source. So, there is less chance of error or distortion. On the other hand, secondary data is collected by the explorer from an indirect source for the purpose of exploration. That is why there is no chance to verify the correctness of the data. As a result, the possibility of data being wrong or distorted cannot be completely ruled out. Hence the reliability of secondary data is less than primary data. But this does not mean that data from secondary sources is less important. Rather, in many cases it is not possible to collect information from primary sources; In that case secondary sources help us. For example, when a newborn baby needs to be diagnosed, the doctor asks various questions to the parent and the child's parent is acting as a secondary source here.

In the cell some properties are given which can be used to check the reliability of a source. When choosing any source, you should consider these properties. Now discuss and solve the following problem in pairs.

## Group Work

Discuss with your classmates to find the answers to the following questions. The box below provides a description of the sources from which scientists gathered information when developing an vaccine. Your task will be to find answers to the following questions by identifying the types and characteristics of data sources.

**Data collection method to check the effectiveness of the vaccine:**

First, scientists developed the vaccine in the laboratory. The vaccine is then applied to different groups of people, to collect data from them about the effectiveness of the vaccine. To develop the vaccine, scientists conducted experiments and collected data in 3 steps. In July 2020, a Phase three trial of the vaccine was conducted. 46,331 people of different ages participated in this test from all over the world. By vaccinating them all, scientists collected data to test its effectiveness. Through careful analysis and testing of the data obtained, the vaccine is proven to be effective.

People with different characteristics participated in the test so that the scientists could make sure that the vaccine worked for different types of people. For example: 49.1% were males and 50.9% were females in this test. Also there were people from various races like Asians, Blacks, Latinos, etc. Participation of people of different age groups was ensured. The number of people according to age is given in the table below:

| Age | Number of participants |
|---|---|
| 12-15 | 2,260 |
| 16-17 | 754 |
| 18-55 | 25,427 |
| 56+ | 17,879 |

a. From what sources did the scientists collect data to make the vaccines? Why?

b. Did all the sources used to collect the information have the same propertiess? If there were differences, describe them.

c. What were the advantages of having age differences among the sources of data collection?

d. In the preparation of this vaccine, data was collected from people of different nations from different parts of the world. Which property of the reliability of the sources is expressed by this?

## Conclusion

In this experience you tried to gain experience in the steps of mathematical exploration or problem solving, discovering patterns through exploration, and checking the reliability of data sources required for exploration. There are some details in it which will become clearer when practiced by hand. Hopefully, in the experiences ahead, you will try to analyze and solve problems with an exploring mind. While this is the end of this experience, let your math study have a new start with mathematical explorations.

# Real numbers in everyday life

### You can learn from this experience

- Square roots of square numbers and cube roots of cube numbers
- Square roots and cube roots of whole numbers
- Properties of square and cube roots
- Square roots of fractions
- Operations with square and cube roots
- Square roots on the number line
- Approximating square and cube roots using the calculator
- The uses of square and cube roots

# Real numbers in everyday life

Every day we use many kinds of numbers for various purposes. How many students are there in your class or school? How many windows are there in your classroom? Counting these things usually require integers. Fractional or decimal numbers are used while measuring height, weight, etc. At times, very large numbers are expressed in exponential form. You are already familiar with fractions, decimal numbers, and exponents. For example, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{4}$, are fractions. 0.25, 3.33, and 5.2555 are decimal numbers and 4^10 is in exponential form. Numbers of these types are called Rational Numbers. There are infinitely many Irrational Numbers as well. In this experience, we will introduce both of these numbers. Rational and Irrational numbers together are called Real Numbers. We come across Real Numbers in our real lives. In this learning experience, we will study the many types of real numbers and their properties.

## Game of divisions

Division of numbers is a part and parcel of our daily activities. For example, sharing 6 slices of bread between 2 people, dividing 100 tk between 5 people, dividing 2 bighas of land between 3 siblings, etc. Doing these calculations are sometimes easy and sometimes difficult. Let's try to understand this using practical examples. Suppose you and your friends buy and eat tiffin together and share the bill equally. The number of friends and the total bills on the respective days are given below. Fill up the table.

**Table-2.1**

| Number of friends | The total cost of tiffin | Cost per person (In fractions) | Cost per person (In decimal form) |
|---|---|---|---|
| 2 | 20 | | |
| 4 | 42 | $\frac{42}{4}$ | |
| 4 | 41 | | |
| 5 | 54 | | 10.80 |
| 3 | 32 | | 10.6666... |
| 3 | 42 | | |
| 6 | 55 | | |
| 7 | 60 | | |

What did you notice while filling up the table above? Write it down:

## Finite and Infinite Decimal Numbers

In the game of divisions, you might have noticed that when you convert from fractions to decimal numbers, sometimes you get an integer, sometimes the digits ended after the decimal and sometimes the digits after the decimal do not end. The numbers whose digits end after the decimal point are called Finite Decimal Numbers. The numbers whose digits do not end after the decimal point are called Infinite Decimal Numbers. If a digit (or group of digits) after the decimal point repeats forever then the infinite decimal number is called a Recurring Decimal/Repeating Decimal. It's not possible to write all the digits of recurring decimals, rather the recurring part is denoted by a " –" or by a "." on it. In the case of "-", it's used on all the recurring digits. In the case of ".", if the number of recurring digits is one or two, it's used on them. If there are more than two recurring digits, "." is used only on the first and last digits. For example:

$33.333\ldots = 33.\dot{3} = 33.\overline{3}$,

$1.2727\ldots = 1.\overline{27} = 1.\dot{2}\dot{7}$,

$0.345345\ldots = 0.\overline{345} = 0.\dot{3}4\dot{5}$

## Rational Number

You might have noticed that a fraction looks like this: $\frac{p}{q}$. A fraction is simply the ratio of two integers. Rational Numbers are those numbers that we can express as a ratio of two integers. A fraction has a numerator and a denominator. Here 'p' is the numerator and 'q' is the denominator. The denominator can't be zero, because that means we have to divide by zero. If you apply the concept of division on a number line, you will see that the value of the ratio becomes undefined.

So,

> The numbers that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are called Rational Numbers.

## Individual Task

Is '0' a rational number? If yes, express it in the form $\frac{p}{q}$

Are the integers, rational? If yes, express any integer in the form $\frac{p}{q}$ .

Are the finite decimals numbers, rational? Express 0.21 and 2.01 in the form $\frac{p}{q}$

Can you tell if a recurring decimal can be expressed as a fraction? Let's give it a try. Here we will be using a bit of algebra.

**Problem:** Express $0.\dot{3}$ = 0.333….. as a fraction.

**Solution:** Let, x = 0.333… ... Then, $10x$ = 3.333... (Multiplying both sides by 10).

Now, subtracting x from 10x, we get:

$$\begin{aligned} 10x &= 3.333\ldots \\ x &= 0.333\ldots \\ \hline (-) \qquad 9x &= 3 \end{aligned}$$

That is,

$$x = \frac{3}{9} = \frac{1}{3}$$

Therefore, $0.333\ldots = \frac{1}{3}$

> 0, the integers and the recurring decimal numbers are all rational numbers.

☞ Keep in mind that an integer can be +ve or –ve. Similarly, a rational number can be +ve or –ve as well.

## Individual Task

1 Express the following decimal numbers into simple fractions:

$0.666\ldots$, $0.4777\ldots$, $1.999\ldots$, $1.\overline{2}7$, $0.23\overline{5}$, $3.0\dot{9}$, $2.\dot{3}\dot{4}$, $0.\dot{1}23\dot{4}$

2 Express the given fractions into decimal numbers and look for a pattern. Then for each of the decimal numbers, convert them into simple fractions using the methods shown earlier. Verify your findings.

$\frac{2}{3}$, $\frac{61}{90}$, $\frac{12}{13}$, $2\frac{34}{99}$

## Approximate value

You wish to divide 100 tk equally between 3 friends. Try to divide it, you will be exhausted. Humans have invented many types of instruments for this purpose. These instruments are our friends in need on different occasions. To calculate things easily and faster, we have made many types of calculators, computers, and digital devices. Use any of these to divide 100 by 3. Can you see the same result on all the devices? Obviously no! "100 divided by 3" is supposed to have one single answer. So which device is giving the correct answer and which one is wrong? Some devices show the answer to be $\frac{100}{3}$. This answer is correct. But when you divide, you see that the digits after the decimal do not end. '3' keeps repeating. But digital devices can only show a fixed number of digits after the decimal point. They can't show any digits after that. So whenever a digital device shows $\frac{100}{3}$ in decimal form, it will only show up to a fixed number of digits.

But if you would have divided 100 by 16 instead of 3, you would get $\frac{100}{16} = 6.25$. Here we have only two digits after the decimal point and no more. In this case, the digital devices would have shown 6.25, which is the exact answer.

Now imagine that a digital device can show the value of $\frac{100}{3}$ up to 1 crore digits after the decimal point. Even in this case, it shows a finite decimal number. And this finite decimal number can never be equal to the exact value of $\frac{100}{3}$ which is 33.33333….., (an infinite decimal number). Therefore, no digital device can accurately give the value of 100/3.

**Now that you know the limitations of digital devices in calculating decimal numbers, would you always trust these devices as your friends? Or you won't trust them at all? If you don't trust them, then it'll be difficult to do large calculations. So what's the solution?**

We have stated in the beginning that digital devices are there for your help like your friends help you in need. But you have to think on your own if it's ok to trust. Similarly, digital devices can not always give the exact answer. They give us answers that are very close to the actual answer. We call these answers "Approximate Values". This is a limitation of digital devices. So we verify our answers by using many devices.

## Equivalent Rational Numbers

In previous classes, we discussed equivalent positive fractions. Here we will discuss both the +ve and –ve fractions. Let's consider the following rational numbers:

$$-\frac{3}{4}, \quad -\frac{6}{8}, \quad -\frac{75}{100}, \quad -0.75$$

Are these numbers different? Use your ideas about fractions and decimal numbers to verify this. I have an idea though.

**Two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal, that is:**

$$\frac{a}{b} = \frac{c}{d} \text{ if } ad = bc.$$

This property of rational numbers is called the Equivalence Property. You might have already found that the numbers mentioned before, all have the same value. That is:

$$-\frac{3}{4} = -\frac{6}{8} = -\frac{75}{100} = -0.75$$

These numbers are all equivalent rational numbers of each other.

## Individual Task

Use the idea of the Equivalence Property of rational numbers to find which of the following fractions are equal:

$\frac{3}{4}$, $-\frac{6}{8}$, $-\frac{75}{100}$, $-0.75$

For any rational number, there are unlimited (infinite) equivalent rational numbers. Multiplying the numerator and denominator of a fraction by any number except '0' gives an equivalent rational number.

For any rational number $\frac{a}{b}$,

$$\frac{ax}{bx} = \frac{a}{b} \ (x \neq 0)$$

$$\text{and } -\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$$

This property of rational numbers is called the Fundamental Property. So, every rational number has infinite number of equivalent rational number.

$$-\frac{3}{4},\ -\frac{6}{8},\ -\frac{75}{100},\ -0.75 \text{ - - -}$$

Consider these numbers again. A property of these numbers is that they are all negative, as they have the negative sign before them. So these are negative rational numbers.

We can simplify fractions using the Fundamental Property. First, we factorize the numerator and denominator into prime factors, then apply the Fundamental Property. For example:

$$\frac{30}{36} = \frac{2 \times 3 \times 5}{2 \times 2 \times 3 \times 3} = \frac{5}{6}$$

## Individual Task

Simplify the following using the Fundamental Property:

$\frac{2}{3}$, $-\frac{3}{2}$, $\frac{6}{9}$, $\frac{16}{24}$, $-\frac{15}{10}$, $-\frac{5}{15}$,

## Pair Work:

Pair up with your friend. Both of you write down two equivalent rational numbers on a paper and show it to each other.

## Square Root

Many of you have a hobby of gardening. Some have a hobby of flower gardens, some have a hobby of vegetable gardens, while others have a hobby of fruit gardens. Many schools have empty spaces. There might be space in your courtyard or on your rooftop. Suppose, you are given some space to make a square-shaped garden of area 16 units squared. Can you tell me what will be the length of the garden? You will surely say the length is 4 units. The reason is:

We know, the area of a square is its length squared. So, if length = 4, then the area is $4^2 = 4 \times 4 = 16$ units squared. Here 4 is called the square root of 16. It's written as $\sqrt{16}$.

Hence,

$$4 = \sqrt{16}.$$

Now tell me, is there any other number whose square is also 16? Write your answer here:

| |
|---|
| |

Let me remind you that, you have studied the multiplication of –ve numbers in your former classes. You should know when and how to use –ve numbers when we need them. Notice that:

$$-4 \text{ squared} = 16$$

Does that mean –4 is also a square root of 16? Yes, it is indeed. So, 16 has two square roots: +4 and –4. But:

$$-4 \neq \sqrt{16}$$

Here, $\sqrt{\phantom{x}}$ gives the +ve square root only. That is:

$$\sqrt{16} = 4$$

So,

$$-4 = -\sqrt{16}$$

We use $\sqrt{\phantom{x}}$ to denote the positive square root of a number. This +ve square root is called the Principal Square Root. The –ve root is represented with $-\sqrt{\phantom{x}}$ on the number.

For example:

$$4 = \sqrt{16} \qquad \text{and} \qquad -4 = -\sqrt{16}.$$

If 'a' and 'b' are two numbers, then 'a' is a square root of 'b' if $a^2 = b$ . Here, 'b' is called the square of 'a', and 'a' is called the square root of 'b'. If 'a' is +ve, then:

$$a = \sqrt{b}$$

Since the square of any number is either '0' or a +ve number, hence '0' and all the +ve numbers have square roots. '0' is the square root of itself, that is, $\sqrt{0} = 0$. All the +ve numbers have two square roots. One of them is +ve, the other is –ve. Now think, how do you find the square root of a –ve number? Write down your ideas below.

## Principal square root of a perfect square

If you square 2 and then take its square root, what do you get? Write your answer below:

Now, if you square –2 and then take its square root, what do you get? It's easy to see that $\sqrt{(-2)^2} = \sqrt{4} = 2$ .

Therefore, by replacing numbers with letters, we can write:

If $a$ is a real number $\sqrt{a^2} = |a|$

Here, $|a| = \begin{cases} -a, & a < 0 \\ a, & a \geq 0. \end{cases}$

$|a|$ is called the Absolute Value of '$a$'.

**Using the formula for Absolute Value we get:**

$$|2| = 2 \quad \text{and} \quad |-2| = -(-2) = 2.$$

## In Search of Irrational Numbers

Suppose you are given some land to plant a square-shaped garden. The land area is 15 square units. So tell me, what's the length of your garden? You answer "Its length is $\sqrt{15}$ units". Now, the question is, is $\sqrt{15}$ a rational number? That is, can $\sqrt{15}$ be written in the form $\frac{p}{q}$ ? To look for an answer, we have to travel back in time, to the era of Pythagoras. So, let's get ready for the adventure.

(Hippasus)

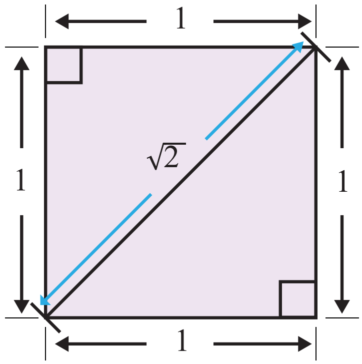


Hippasus was a follower of the Greek mathematician Pythagoras. One day Hippasus thought that if the length of a square is 1 unit (1 unit can be 1m or 1 cm or 1 inch or 1 of any other unit), then what's the length of the square's diagonal? Now, if we cut a square along its diagonal, we get a right-angled triangle with equal base and height. In other words, we can ask: If a right-angled isosceles triangle has a base and a height of

1 unit, what is the length of its hypotenuse?

Using Pythagoras' theorem, we get:

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$




So, Hypotenuse $= \sqrt{(\text{Base})^2 + (\text{Length})^2} = \sqrt{1^2 + 1^2}$ unit $= \sqrt{2}\ units.$

Is this number rational? Hippasus claimed that it's not rational. That means, $\sqrt{2}$ can not be expressed in the form $\frac{p}{q}$. Hippasus proved this fact, and it was the first time humans got to know about a number that is not rational. You will study this proof in later classes. These types of numbers are called Irrational Numbers. Your garden measures $\sqrt{15}$ in length, this is also an irrational number like $\sqrt{2}$. You can see that these numbers come from real-life problems. Therefore, these are real numbers. Now, what's the value of this type of number, and how to find this value are important for us to know.

It should be mentioned that, among the natural numbers, only the square root of square numbers are rational numbers. If it's not a square number, then its square root is an irrational number.

In general, if we remove the rational numbers from the world of real numbers, then the remaining numbers are the irrational numbers.

## Calculating the square root of a number

We have learned how to find the square roots of square numbers. Such square roots are rational numbers. But if it's not a square number, then the square root is an irrational number. In that case, how do we find the square root? Let's find the value of $\sqrt{2}$ using the division method up to a certain number of decimal places.

| | | |
|---|---|---|
| | | 1. 4 1 4 2 |
| | 1 | 02. 00 00 00 00... |
| $1 \times 1 \rightarrow$ | | −1 |
| | 24 | 100 |
| $24 \times 4 \rightarrow$ | | −96 |
| | 281 | 400 |
| $281 \times 1 \rightarrow$ | | −281 |
| | 2824 | 11900 |
| $2824 \times 4 \rightarrow$ | | −11296 |
| | 28282 | 60400 |
| $28282 \times 2 \rightarrow$ | | −56564 |
| | | 3836 |
| | | ... |

Table-2.2

For example, if we want to approximate the value up to 4 decimal places, we will be taking the intermediate values up to at least 8 decimal places. We will be placing '0's on the right as necessary. In the image, the value of $\sqrt{2}$ has been found up to 4 decimal places by the division method.

## Individual Task

1. Find the value of $\sqrt{2}$ by the division method up to 6 decimal places.
2. Use a digital device to find the value of $\sqrt{2}$. Compare this with the value you found in question 1. Is there any difference? If there's a difference, then correct the mistake.
3. Find the value of $\sqrt{15}$ by the division method up to 4 decimal places.
4. Use a digital device to find the value of $\sqrt{15}$. Compare this with the value you found in question 3. Is there any difference? If there's a difference, then correct the mistake.
5. By using the division method, and with the help of a digital device, find the square roots of five more numbers that are not square numbers.

## Property of principal square roots

We can simplify square root of integer using characteristics of square root. Some characteristics of square root are given bellow.

If '$a$' and '$b$' are two numbers either 0 or greater than 0, then:

$$\sqrt{a}\sqrt{a} = a, \qquad (-\sqrt{a})(-\sqrt{a}) = a, \qquad \sqrt{ab} = \sqrt{a}\sqrt{b}$$

**Example:** $\sqrt{36} = \sqrt{2 \times 2 \times 3 \times 3} = \sqrt{2 \times 2}\,\sqrt{3 \times 3} = (\sqrt{2}\,\sqrt{2})\,(\sqrt{3}\,\sqrt{3}) = 2 \times 3 = 6$

## Property of principal square roots of common fractions

If 'a' is 0 or any +ve integer, and '$b$' is a +ve integer, then

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

**Example:** Find the square root of $\frac{9}{16}$

**Solution:** Using the above property, we have: $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{\sqrt{3 \times 3}}{\sqrt{4 \times 4}} = \frac{\sqrt{3}\,\sqrt{3}}{\sqrt{4}\sqrt{4}} = \frac{3}{4}$

So, the square root of $\frac{9}{16} = \pm\, \frac{3}{4}$

**Example:** Find the square root of $\frac{27}{48}$

**Solution:** $\sqrt{\frac{27}{48}} = \sqrt{\frac{3 \times 9}{3 \times 16}} = \sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

So, the square root of $\frac{27}{48} = \pm\, \frac{3}{4}$

Notice that both the numerator and denominator of $\frac{9}{16}$ are square numbers. Also, in the case of $\frac{27}{48}$ , when expressed in simplest form, both its numerator and denominator are square numbers. Fractions like this are called Perfect Square Fractions.

**Example:** Find the square root of $\frac{9}{32}$

**Solution:** $\sqrt{\frac{9}{32}}=\frac{\sqrt{9}}{\sqrt{32}}=\frac{\sqrt{3\times3}}{\sqrt{2\times2\times2\times2\times2}}=\frac{3}{4\times\sqrt{2}}=\frac{3\times\sqrt{2}}{4\times\sqrt{2}\times\sqrt{2}}=\frac{3\times\sqrt{2}}{4\times2}=\frac{3\sqrt{2}}{8}$,

So, the square root of $\frac{9}{32}=\pm\frac{3\sqrt{2}}{8}$

Notice, here that numerator of $\frac{9}{32}$ is 9, which is a perfect square. But the denominator is 32, which is not a perfect square. Also, we cannot simplify $\frac{9}{32}$ any further. Hence, $\frac{9}{32}$ is not a perfect square fraction.

## Individual Work

Use a digital device and also use the methods you've learned to find the square roots of the common fractions: $\frac{32}{50}$, $\frac{41}{441}$, $\frac{1089}{121}$. Verify if these are perfect square fractions.

Compare the answers you found using both ways and give your opinion on them.

## Finding the square roots of decimal fractions

In your previous classes, you have learned how to add, subtract, multiply, and divide decimal fractions. You have also learned the technique of converting decimal fractions into common fractions and then doing addition, subtraction, multiplication, and division on them easily. We will use these techniques to find out the square roots of decimal numbers.

**Example:** Find the square root of 1.2.

**Solution:** $\sqrt{1.2} = \sqrt{\frac{12}{10}} = \frac{\sqrt{6}}{\sqrt{5}} = \frac{\sqrt{6}\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{\sqrt{30}}{5}$

Now, using the division method, we find the value of $\sqrt{30}$ in decimal form. And divide this value by 5 to get the value of $\sqrt{1.2}$ in decimal form.

## Individual Task:

Use a digital device and also use the methods you've learned to find the square roots of 0.25, 0.0001 and 10.24. Compare the answers you found using both ways and give your opinion.

## Irrational Numbers on the number line

In the earlier discussions, you have learned about number lines and also the Theorem of Pythagoras. Now we will talk about where irrational numbers lie on the number line.

## The positions of $\sqrt{2}$ and $\sqrt{3}$ on the number line

Let O be the origin on the number line. Let B be a point such that B is 1 unit to the right of O and A is 1 unit directly above the number line from B. What's the length of OA? Using Pythagoras' Theorem,

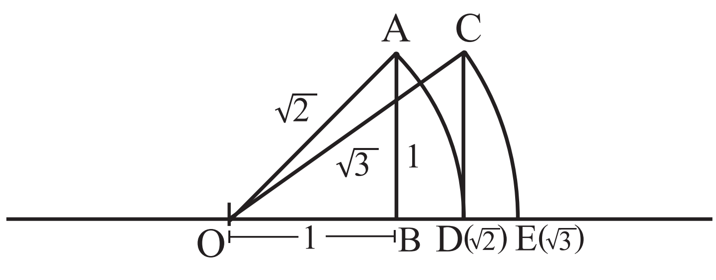



Figure-2.2

$OA = \sqrt{1^2 + 1^2} = \sqrt{2}$

We choose point D on the right side of the number line so that OA and OD are equal in length. It's easy to see that point D is the position of $\sqrt{2}$ on the number line.

Now choose point C so that DC is perpendicular to OD and DC = 1. What's the length of OC? Use Pythagoras' Theorem you can show that OC = $\sqrt{3}$ . Let's take a point E on the number line on the right side of O so that OE = OC. Note that point E is the position of $\sqrt{3}$ on the number line.

☞ Notice that, although we can not find the exact values of $\sqrt{2}$ and $\sqrt{3}$ we can locate them correctly on the number line.

## Individual Task

Find the positions of $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$,… on the number line as discussed above.

## Cube Root

By now you know about square roots. Have you heard about cube roots? We know that the square root is the opposite of the square. Similarly, the cube root is the opposite of the cube. Using variables, we say that 'a' is the cube root of 'b' if:

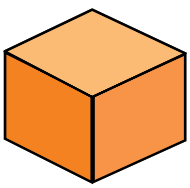
$$a^3 = b$$

We use the sign $\sqrt[3]{\ }$ to express cube roots. Since 64 is the cube of 4, so

$4^3 = 4 \times 4 \times 4 = 64$. Therefore

$$\text{Cube root of } 64 = \sqrt[3]{64} = 4$$

While using square roots, we saw that the square root is the length of a square. Similarly, the cube root is the length of a cube.



A cube has sides of length x metres and volume 64 $m^3$ . Thus: $x \cdot x \cdot x = 64$, or, $x^3 = 64$ or, $x = \sqrt[3]{64} = 4$.

Now think, if $\sqrt[3]{64} = 4$ then what does $\sqrt[3]{-64}$ indicate?

That is, what number would be –64 after cube? you will surely say, –4,

because $(-4) \times (-4) \times (-4) = -64$.

i.e; $(-4)^3 = -64$

So, We can say,

if $x^3 = y$, then $x = y^{\frac{1}{3}}$, where $x$ may be positive or negative.

## Properties of cube roots

By using the properties of cube roots we can simplify expressions involving cube roots of integers. Below are two such properties:

If '$a$' and '$b$' are integers, then:

$$\sqrt[3]{a}\ \sqrt[3]{a}\ \sqrt[3]{a} = \left(\sqrt[3]{a}\right)^3 = a \qquad \sqrt[3]{ab} = \sqrt[3]{a}\ \sqrt[3]{b}$$

## Finding cube roots by prime factorization

Similar to how we evaluated square roots, we evaluate cube roots by expressing the number in terms of its prime factors. For example, for the number 216:

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$$

Here, there are three 2s and three 3s. While evaluating square roots, we took pairs of the same factor. While evaluating cube roots, we take triplets of the same factor. For example, cube root of 216 = 2 × 3 = 6, so:

$$\sqrt[3]{216} = 2 \times 3 = 6$$

## Individual Work

With the help of prime factors or divisors, find the cube roots of the following:

12161, –9261, 15625, –262144, 9261000, 32768

## Cube roots of common fractions

Let's see how to find the cube roots of fractions. Here we can use the ideas of cubes and cube roots.

Cube root of $\dfrac{64}{125} = \sqrt[3]{\dfrac{64}{125}} = \sqrt[3]{\dfrac{4\times4\times4}{5\times5\times5}} = \sqrt[3]{\left(\dfrac{4}{5}\right)^3} = \dfrac{4}{5}$

Again,

$$\frac{\sqrt[3]{64}}{\sqrt[3]{125}} = \frac{4}{5} = \frac{\sqrt[3]{4^3}}{\sqrt[3]{5^3}}$$

Thus, it can be seen that the cube root of a fraction is the cube root of the numerator divided by the cube root of the denominator. This is similar to square roots of common fractions.

Notice that, here the numerator and denominator are both +ve numbers. If one of them is –ve, then how do we find the cube root?

## Properties of cube roots of common fractions

If '$a$' is any integer, and '$b$' is any integer except 0, then:

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

## Individual Work

Find the cube roots of the following fractions: $\dfrac{27}{216}$, $\dfrac{704969}{35937}$, $\dfrac{13824}{166375}$,

## Arithmetic of square and cube roots

Just like rational numbers, we can add, subtract, multiply and divide square or cube roots.

## Addition or Subtraction

While adding or subtracting, we must have the square/cube root of the same number. The coefficients are then added or subtracted accordingly.

**Example:** Find out: i) $5\sqrt{2} + 2\sqrt{2}$ , ii) $5\sqrt{2} - 2\sqrt{2}$

**Answer:**

$5\sqrt{2} + 2\sqrt{2} = (5+2)\sqrt{2} = 7\sqrt{2}$

$5\sqrt{2} - 2\sqrt{2} = (5-2)\sqrt{2} = 3\sqrt{2}$

## Multiplication or division

We can multiply or divide square/cube roots by using the properties of square and cube roots. We also need to know how to multiply/divide rational numbers.

**Example:** Multiply $\sqrt{2} + \sqrt{3}$ with $\sqrt{2} - \sqrt{3}$ .

**Answer:**

$(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) = \sqrt{2}\sqrt{2} + \sqrt{2}\sqrt{3} - \sqrt{2}\sqrt{3} - \sqrt{3}\sqrt{3} = 2 - 3 = -1$

What you observe in this example, write it down below. Here, $\sqrt{2} - \sqrt{3}$ is called the Conjugate of $\sqrt{2} + \sqrt{3}$. The opposite is also true.

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**We need to get rid of the square root in the denominator when dividing.**

**Example:** Divide $\sqrt{2}+\sqrt{3}$ by $\sqrt{2}-\sqrt{3}$.

**Solution:** $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})\,(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})\,(\sqrt{2}+\sqrt{3})} = \cdots = -(5+2\sqrt{6})$ [Do the calculations in-between]

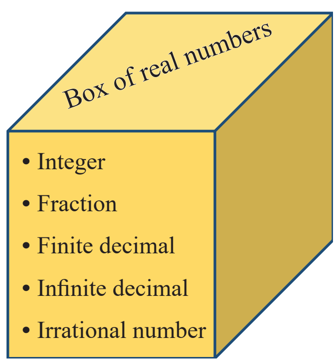
## Individual Task

- Work out: i) $2\sqrt{2}+\sqrt{8}$ ii) $2\sqrt{2}-\sqrt{8}$
- Work out: i) $\sqrt[3]{2}+\sqrt[3]{54}$ ii) $7\sqrt{2}-\sqrt[3]{54}$
- Do addition, subtraction, multiplication and division with the numbers $2\sqrt{3}+5\sqrt{2}$ and $7\sqrt{2}-4\sqrt{3}$. Then represent the answers on a number line.
- Rationalise the denominator (Get rid of the square root in the denominator) of $\frac{5}{\sqrt{3}+\sqrt{5}}$
- Find the value of x: $\frac{1}{\sqrt{19}-\sqrt{12}} = \frac{\sqrt{19}+\sqrt{x}}{7}$
- Simplify: $\frac{1}{9+\sqrt{3}} + \frac{1}{3+\sqrt{2}}$

**The sum, difference, product or quotient of two irrational numbers may be rational or irrational.**

## Box of real numbers

Let's write down different real numbers on pieces of paper, and put these pieces in a box. We name it: ''Box of real numbers''. Now take some pieces out randomly and note down the type of number in the table below. Have you found any number you don't know of?




2, $\frac{3}{5}$, 2.34, $\pi$, $\sqrt{2}$

| Table 2.2 | | | | | |
|---|---|---|---|---|---|
| | **Integer** | **Fraction** | **Finite decimal** | **Infinite decimal** | **Irrational number** |
| 2 | | | | | |
| | | | | | |
| $\frac{3}{5}$ | | | | | |
| | | | | | |

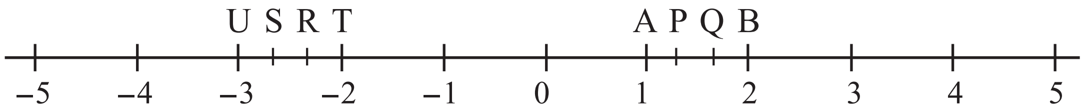
## Exercise

1. Long jump is a fun game in sports competitions. Suppose you are to jump and touch a wall 10 m away. However, you can only jump halfway to the goal in every jump. For example, in the first jump, you cross $\frac{10}{2} = 5$m. In the next jump, you cross $\frac{5}{2} = 2.5$m, and so on. Can you find many how many jumps you need to get to the wall in this way?

2. In a square-shaped mango garden there are 1369 mango trees. If there is an equal number of mango trees along the length and breadth of the garden, then what's the number of mangoes in each row? Give logical reasons for your answer. If the distance between any two trees is 100 ft, then what's the approximate area of the garden?_

3. Find the square root of all the perfect squares from 1 to 100. Also, find the cube root of all the perfect cubes from 1 to 100.

4. On a number line the points P, Q, R, S, T, U, A and B are located such that TR = RS = SU and AP = PQ = QB. Find the values of the rational numbers P, Q, R and S.




5. Are the numbers below rational or irrational? Explain.

   8.929292…, 0.1010010001…, 6534.789749…, 2.18281828, 0.122333…

6. Perform addition, subtraction, multiplication and division with the numbers $2\sqrt{2} + 5\sqrt{8}$ and $7\sqrt{8} - 4\sqrt{2}$ . Hence, represent the answers on a number line.

7. Simplify: $\sqrt[3]{\frac{3}{5}} + \frac{\sqrt[3]{9}}{5} - \sqrt[3]{81}\ 1$

8. Nisith Chakma has two square-shaped vegetable gardens.

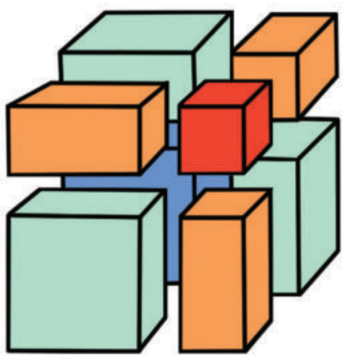
   The length of one garden is $2\sqrt{2}$ units, and the area of the other is twice the area of the first. What's the length of the other garden?

9. You have two cube-shaped boxes. One box has a volume of 16 $m^3$ and the other has a volume of 11 $m^3$. What are the lengths of the sides of each box? If the boxes are split and made into a single cubic box of volume equal to the sum of the volumes of the two boxes, then what's the length of the sides of the new box?

# Finding binomial and trinomial expressions using solids

## You can learn from this experience

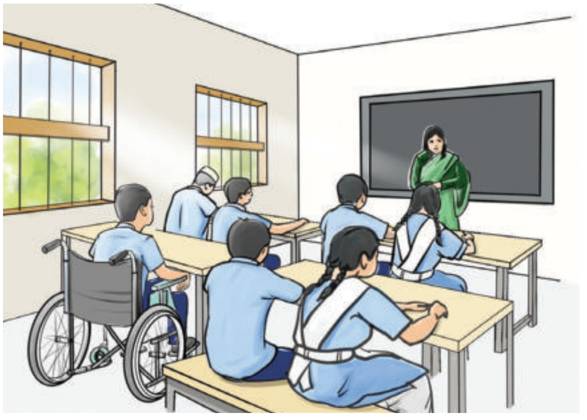
- Finding algebraic relations by observing patterns in mathematical expressions.
- Finding real or abstract patterns from mathematical relations.
- Expressing mathematical relations using signs and variables.
- Establishing relations between geometric sizes and mathematical expressions.
- Using mathematical expressions to solve real life problems.



# Finding binomial and trinomial expressions using solids

In your previous classes you experienced the uses of variables, algebraic expressions, terms, factors of algebraic expressions, LCM, GCD/HCF, etc. Algebraic expressions play a vital role in solving real life problems. You learnt binomial and trinomial expressions regarding squares and rectangles. You know that rectangle is a two-dimensional size. It means there are two dimensions of measuring this, length and width. Also, square is a special case of rectangle. The length and width of a square are equal. A fun fact is, three dimensional objects are more in number around us than the two dimensional objects. For example – books, writing pads, cupboards, showcases, bookshelves etc. In a three-dimensional object, there is an additional dimension with length and width, which is – height. Two dimensional objects with length and width are called rectangular. In a similar way, three dimensional objects with length, width and height are called cubic. In this experience we will learn to use binomial and trinomial expressions using solids. First, let's learn how to construct binomial expressions using two dimensional objects.

## Looking for binomial expressions using classroom windows



Notice the windows of your classroom, it has length and width. Find out the relation between the length and the width. Suppose my window is 5 feet long and 3 feet wide. Then, length is 2 feet more than width. It means,

Length = Width + 2 feet = (3+2) feet

This is one relation between the length and the width. We can express it in a different way, too. Here the length is 1 foot less than the double of the width. It means,

Length = (2× Width – 1) feet = (2 × 3 – 1) feet = (6 – 1) feet

Here, 3 + 2 and 6 – 1 are two binomial expressions of numbers.

## Concept of algebraic binomial expressions from numerical binomial expressions

Suppose you don't know the length and width of a classroom window but you know about their relation. The relation is – length is 2 feet more than width. That is,

Length = Width + 2 feet

Here length is dependent on the width. To show this relation, we will use a variable. If the width is x feet, then

Length = (x + 2) feet

Here (x + 2) is a binomial expression. In the same way if an expression has two terms, then it is called a binomial expression. In this expression x + 2, there is only one variable, x. Hence it is a single variable binomial expression. The binomial expressions having only one variable are called single variable binomial expressions.

## Finding binomial expression in shrimp pond


Now we will get the idea about binomial expressions with two variables. Suppose Rayhan's father has two rectangular shrimp ponds, A and B (See the figure). The length of pond B is equal to the sum of length and width of pond A. Here both length and width of pond A are unknown variables. Hence we need to use two variables. Let the length of A is x and width of A is y. Then the length of pond B is x+y. Which means the length of B is a binomial expression with two variables. Similar binomial expressions having two variables are called binomial expressions with two variables.

### Individual Task:

Write down 5 binomial expressions on your own and present them with real life examples.

## Group Work:

Divide your class roll by 4. The remainder is your team number. For example, if the remainder is 0 then your team number is 0. In the same way there will be four teams in total, team-0, team-1, team-2 and team-3. Now add 1 and 5 with your team number, separately. You will get two new numbers. From the exercises below, present the expressions corresponding to those numbers using real life examples.

1) $x + 3$ 2) $2x + 1$ 3) $3x - 3$ 4) $x - 2$

5) $5x + y$ 6) $x^2 - 1$ 7) $x^2 - y$ 8) $x + y^2$

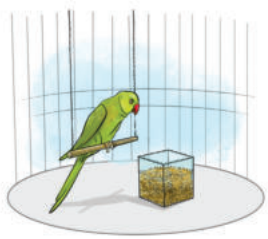
Now to evaluate other team's works, fill out the following table and hand it to your teacher.

**Tabel 3.1**

| Evaluation table of team-0 | Binomial expression | Evaluation of presented real life example | Reason behind your evaluation | Present one different real life example from your team |
|---|---|---|---|---|
| Team-1 | 2) | | | |
| | 6) | | | |
| Team-2 | 3) | | | |
| | 7) | | | |
| Team-3 | 4) | | | |
| | 8) | | | |

## Cube and Rectangular Solid

In class 7 you learnt about the size and volume of cubes and rectangular solids. They are three dimensional. If we add one more dimension with rectangle, we get the rectangular solids. For example, the floor of your classroom has length and width. If we add the third dimension, height, we get the rectangular classroom. Fun part is, if we reduce one dimension from a solid, we again get a rectangular area. For example, if we discard the height of the classroom, we will obtain the floor. Can you tell what will happen if you discard the length? Again, what will happen if you discard width? As we have mentioned earlier, there are more three-dimensional objects than two-dimensional surrounding us. In the figure, the bird feeding box is a cube and the brick is a rectangular solid.



### Volume

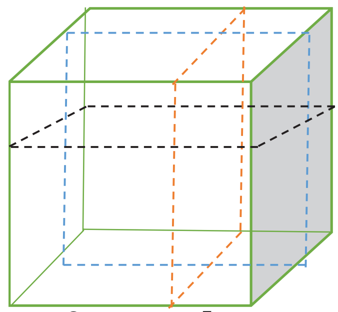
The length, width and height of a cube are equal, and we write the size of the cube to be length × length × length. Here volume of the cube = $(\text{length})^3$. It means if the length of the cube = $l$ , then the volume V = $l^3$.

For a rectangular solid, the length, width and height are not always equal. The size of a rectangular solid is length × width × height. For a rectangular solid, if length = $l$, width = $w$ and height = $h$, then, the volume of the rectangular solid V = $lwh$.

## Finding cube of a binomial expression while making a showcase

Suppose you want to build a cube shaped glass showcase at your home which is open in all side. There will be shelves of various sizes. Let's make a design of the showcase on paper.

**Example -1** you will put a glass in the middle along the length, width, and height of the showcase such that on one side there will be 8 units and on the other side there will be 5 units (like the figure on the right). The dotted lines in the figure represent the glasses placed in the middle.




- Write down the size of your showcase and find out the volume.
- How many partitions of the showcase will be formed?
- Label each part using $V_1$, $V_2$, $V_3$, … etc.
- Find out the length, width and height of each partition.
- Write down the size of each partition and find out the volume of each.
- Find out the number of partitions having same size.
- Is there any relation between volume of the showcase and volumes of the partitions? If you find something write down.

Fill out the following table using the answers from the questions above.

**Table 3.2**

| Showcase | length, width and height | Size | Volume |
|---|---|---|---|
| | 8 + 5, 8 + 5, 8 + 5 | | $(8 + 5)^3$ |
| **Partition label** | **length, width and height of the partition** | **Size of partition** | **Volume of partition** |
| $V_1$ | 8, 8, 8 | | |
| $V_2$ | 8, 5, 8 | | |
| $V_3$ | 8, 5, 8 | | |
| $V_4$ | 5, 5, 8 | | |

| | | | |
|---|---|---|---|
| $V_5$ | 8, 8, 5 | | |
| $V_6$ | 8, 5, 5 | | |
| $V_7$ | 8, 5, 5 | | |
| $V_8$ | 5, 5, 5 | | |

Volume of the showcase, $V = (8 + 5)3 = 133 = 2197$

Sum of volumes of the partitions,

$$=8^3 + (8^2 \times 5) + (8^2 \times 5) + (8^2 \times 5) + (8 \times 5^2) + (8 \times 5^2) + (8 \times 5^2) + 5^3$$

$$=8^3 + 3 \times (8^2 \times 5) + 3 \times (8 \times 5^2) + 5^3$$

$$=512 + (3 \times 320) + (3 \times 200) + 125$$

$$=512 + 960 + 600 + 125 = 2197$$

☞ Notice, the volume of the showcase and total volume of the partitions are equal. So from this relation we can write,

$$(8 + 5)^3 = 8^3 + 3 \times (8^2 \times 5) + 3 \times (8 \times 5^2) + 5^3 \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{(i)}$$

**Example-2** imagine you are putting a glass in the middle along the length, width, and height of the showcase such that on one side there will be 7 units and on the other side there will be 6 units. Then what would be the relation between the volume of the showcase and total volume of the partitions? Of course you can answer this,

$$(7 + 6)^3 = 7^3 + 3 \times (7^2 \times 6) + 3 \times (7 \times 6^2) + 6^3 \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{(ii)}$$

## Pair Work

Keeping the size 13 × 13 × 13 fixed, can you express the above relation using two other numbers? (Hint: Two numbers can be 9,4. Write similar example for at least two more pairs of numbers.)

Write down the properties you have noticed from the previous two examples in your notebook. Discuss with your classmate to logically understand different properties they noticed. Take help from your class teacher.

## Observing patterns

Pattern is a very important topic in mathematics. Do you know what the mathematicians and scientists do before proposing and proving a theory? They observe some incident regarding their topic very carefully. Then they try to find out some pattern from the series of incidents. They propose a theory relating to the pattern. Finally, they prove that theory using mathematical reasoning or experiments in laboratories. Like a mathematician we will observe the pattern of the relations of volumes of solids and find out and prove the formula for cubing a binomial expression.

☞ Notice that, according to the relation (i) we can write,

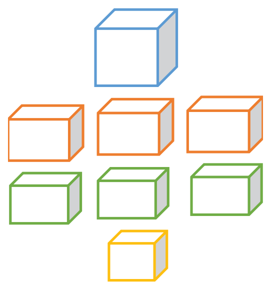
Partitions of size $8 \times 8 \times 8 = 1$

Partitions of size $8 \times 8 \times 5 = 3$

Partitions of size $8 \times 5 \times 5 = 3$

Partitions of size $5 \times 5 \times 5 = 1$

Total partitions = 8



## Individual task

According to the relation (ii),

- Write the size of the partitions, number of partitions of different sizes and total number of partitions.
- Write the size of the partitions, number of partitions of different sizes and total number of partitions for the relations found by yourself.

## Formula for cube of a binomial expression

Observing the pattern of the numerical expression above, using a and b as variables for the two numbers, we can write

$$\boxed{(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3}$$

This is a formula for cube of a binomial expression. We can prove this in several ways. Proof using algebraic calculation: From rules of exponents we get,

$$(a + b)^3 = (a + b)\ (a + b)(a + b)$$

$$=(a + b)\ (a^2 + ab + ab + b^2\ )$$

$$=(a+b)\ (a^2 + 2ab + b^2\ )$$

$$=a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$=a^3 + 3a^2b + 3ab^2 + b^3$$

## Geometric proof (game of eight solids): :

Here we will prove the formula using eight solids like working in a laboratory. To do that, take a stick with suitable size.

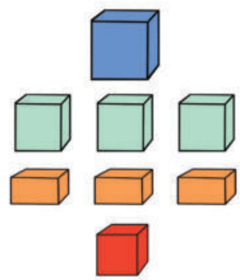

Mark the stick in the middle (like the figure) using a pencil. Name one side as *a* and the other side as b. Now using clay, hardboard, or cork sheet or any suitable material, make eight solids of size length×width×height .

1 solid of size a×a×a

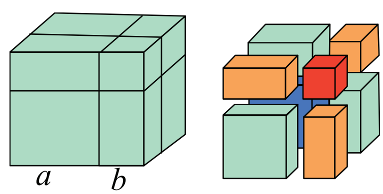
3 solids of size a×a×b

3 solids of size a×b×b

1 solid of size b×b×b



Now perform the following tasks.




- Arrange the solids in such a way that they make a cube.
- Does the length of the cube match with the stick you picked? If not, there is some error in making of your solids. Correct the errors.
- Using the matching of the cube with the length of the stick, measure the length and volume of the cube.
- Find the total volume using the volume of the eight solid parts.
- Write the relation of the volume of the cube with the volume of the parts mathematically. You will get the formula. If not, there is some error in your calculations. Discuss with your classmates or teacher to find out your error.

## Formula for cube of subtraction of binomial expressions

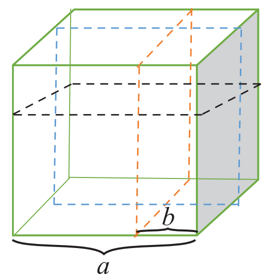
Suppose, the length of sides of a cube is a units. If we subtract b units from each side, we will have eight solids of size length × width × height in the following way

1 solid of size $(a - b) \times (a - b) \times (a - b)$

3 solids of size $(a - b) \times (a - b) \times b$

3 solids of size $(a - b) \times b \times b$

1 solid of size $b \times b \times b$




According to our condition, total volume of these eight solids will be equal to volume of the cube = $a^3$

That is,

$$a^3=(a - b) (a - b) (a - b) + 3(a - b) (a - b) b + 3( a - b) b^2 + b^3$$

$$=(a - b)^3 + 3 (a - b)b (a - b + b) + b^3$$

$$=(a - b)^3 + 3ab (a - b) + b^3$$

$$=(a - b)^3 + 3a^2b - 3ab^2 + b^3$$

Hence,

$$\boxed{(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3}$$

This is the formula for cube of subtraction of binomial expressions.

## Finding other formulae from the formulae of cube

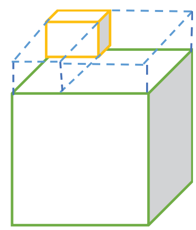
Main two formulae for cube of binomial expression is:

1) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
2) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

## Individual task 4

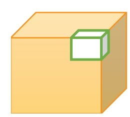
1. Using algebraic rules, construct the following formulae using the main two formulae given above.

| | |
|---|---|
| 3) | $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ |
| 4) | $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ |
| 5) | $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ |
| 6) | $(a - b)^3 = a^3 - b^3 - 3ab(a - \mathrm{b})$ |
| 7) | $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$ |
| 8) | $a^3 - \mathrm{b}^3 = (a - b)\ (a^2 + ab + b^2)$ |



2. Prove the formulae 4) and 5) using the idea of solids. (In the figure on the right the idea of solid is given)

3. Prove the formulae 7) and 8) using the idea of solids. (In the figure on the right the idea of solid is given)



## Using the cubing formula of binomial expressions

**Problem 1.** Calculate the value of $(102)^3$ using cubing formula of binomial expressions.

**Solution:**

$(102)^3 = (100 + 2)^3$

$=100^3 + 3 \times 100^2 \times 2 + 3 \times 100 \times 2^2 + 2^3$ [Accorading to formula 1]

$=1000000 + 3 \times 10000 \times 2 + 3 \times 100 \times 4 + 8$

$=1000000 + 60000 + 1200 + 8$

$=1061208$

**Problem 2:** Calculate the cube of $2x - y$ using cubing formula of binomial expressions.

**Solution:**

In main formula (2), we put $a = 2x,\ b = y$ to obtain,

$$(2x - y)^3 = (2x)^3 - 3(2x)^2y + 3(2x)\,y^2 - y^3$$

$$= 8x^3 - 12x^2y + 6xy^2 - y^3$$

## Individual work:

1) Calculate the following values using cubing formula of binomial expressions.

   i) $(52)^3$ ii) $(79)^3$

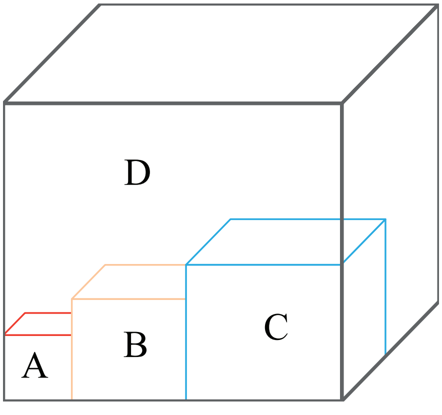
2) Calculate the cube of following binomial expressions using formula.

   i) $x + 1$ ii) $x - 3$ iii) $3x + 5$ iv) $5x - 3$

   v) $2x + 3y$ vi) $x^2 + 1$ vii) $x^2 - y$ viii) $x^2 + y^2$

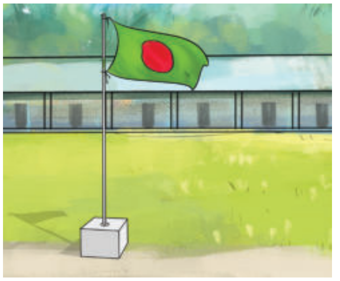
## Cube of Trinomial Expression

### Trinomial Expression




In the given figure, there are four cube shaped boxes, A, B, C and D . Length of each side of cube D is equal to sum of lengths of sides of cubes A, B, C. Hence the length of sides of D are dependent on the sides of A, B, C, where the lengths of sides of A, B, C are unknown. Suppose, the length of sides of boxes A, B, C are respectively $x, y, z$ units. Then the length of each side of D is $x + y + z$ units. Here $x + y + z$ is an algebraic expression with 3 terms. This type of algebraic expressions having 3 terms are called **trinomial expressions**. Here $x + y + z$ is a trinomial expression with 3 variables.

## A real life example:



The length of the cubic water tank of your school is sum of base, width and height of the flag stand. This means the length of the water tank depends on the base, width and height of the flag stand. If the base, width and height of the flag stand are respectively $x$, $y$ and $z$, what is the length of the water tank. The answer is obviously $x + y + z$. This is a trinomial expression.

## Invividual Task

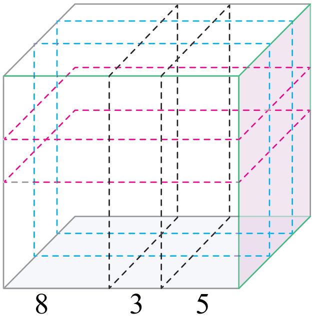
1. Write 10 trinomial expressions on your own. Express 2 out of them using real life example.
2. Which of the following are not trinomial expression. Explain the reason for your answer.

   i) $xy + 3y$     ii) $xy$     iii) $x + y - 1$

   iv) $x^2 - 2x + 1$     v) $xy^2z$

## Cube of a trinomial expression

Let's design a model cube on paper like the cube for binomial expressions. Here each side of the cube has two shelves like the given figure. Suppose the partitions for each side are 8 inch, 3 inch and 5 inch respectively. Now think about the following questions and then answer them.




- Write down the size of the cube and find out the volume.
- Can you tell how many partitions are there in the cube?
- Find out the size of each partition.
- Find out the number of partitions of same size and write down their sizes.
- Find the volume of each size of partitions.
- Is there any relation between the volume of the cube and total volume of the partitions? If yes, write down the relation.

- Fill out the following table using the answer to the above questions.

| Table 3.3 | | | |
|---|---|---|---|
| **Size of cube** | **Number** | **Volume** | |
| $16 \times 16 \times 16$ | | $(8 + 3 + 5)^3$ | |
| **Size of partition** | **Number** | | |
| $8 \times 8 \times 8$ | 1 | $8^3$ | |
| $3 \times 3 \times 3$ | | $3^3$ | |
| $5 \times 5 \times 5$ | | $5^3$ | |
| $8 \times 8 \times 3$ | 3 | $3 \times 8^2 \times 3$ | |
| $8 \times 8 \times 5$ | | | |
| $8 \times 3 \times 3$ | | | |
| $8 \times 5 \times 5$ | | | |
| $3 \times 3 \times 5$ | | | |
| $3 \times 5 \times 5$ | | | |
| $8 \times 3 \times 5$ | 6 | $6 \times 8 \times 3 \times 5$ | |

So, volume of the cube, $V = (8 + 3 + 5)^3 = 16^3 = 4096$

and the sum of volumes of the partitions

$$= 8^3 + 3^3 + 5^3 + (3 \times 8^2 \times 3) + (3 \times 8^2 \times 5) + (3 \times 8 \times 3^2) +$$
$$(3 \times 8 \times 5^2) + (3 \times 3 \times 5^2) + (3 \times 3^2 \times 5) + (6 \times 8 \times 3 \times 5)$$
$$= 512 + 27 + 125 + 576 + 960 + 216 + 600 + 225 + 135 + 720$$
$$= 4096$$

Since volume of the cube and sum of volumes of the partitions are equal, we can write

$$(8+3+5)^3 = 8^3 + 3^3 + 5^3 + (3 \times 8^2 \times 3) + (3 \times 8^2 \times 5) + (3 \times 8 \times 3^2) +$$
$$(3 \times 8 \times 5^2) + (3 \times 3 \times 5^2) + (3 \times 3^2 \times 5) + (6 \times 8 \times 3 \times 5)$$

## Individual task

Write down the size of all partitions to construct the cube whose sides are equal to the sum of the following numbers? How many partitions will be there of each size?

1) 5, 3, 2 2) 8, 5, 3 3) 13, 8, 5 4) 5, 7, 12 5) 6, 4, 2

## Formula for cube of trinomial expressions

Like the cubic expressions, if we use the variables a, b and c for three numbers, observing the pattern from the above numberical expressions we can write,

$$(a+b+c)^3 = a^3+b^3+c^3+3a^2b+3ab^2+3b^2c+3bc^2 + 3a^2c+3ac^2+6abc$$

This is a formula for cube of trinomial expressions. We can prove this in various ways.

## Proof by algebraic rules:

Using the rules of exponents we get,

$$\begin{aligned}(a+b+c)^3 &= (a+b+c)(a+b+c)(a+b+c)\\ &= (a+b+c)(a^2+b^2+c^2+2ab+2bc+2ac)\\ &= a^3+b^3+c^3+3a^2b+3ab^2+3b^2c+3bc^2\\ &\quad +3a^2c+3ac^2+6abc\end{aligned}$$

## Proof by solids

Now we will make some solids of required size to prove the formula for cube of trinomial expressions.

## Group Work

Create four teams, team-0, team-1, team-2 and team-3 as before. Each team will make required number of solids and then putting them together to make a cube.

## Name of the project

Finding the cube of a trinomial expression using solids.

**Required elements:** A stick of suitable length (the length of our intended cube), pencil, clay or hard paper or piece of wood or cork sheet or any suitable element to help the teamwork.

## Working procedure


Teacher will bring a stick of suitable length to class. Each team will take a stick of same length and mark that in two places. Put $a$, b and c in three parts of the stick (like the figure). Each team will have marking in different places. Now break the sticks in three parts along the markings. Now using the element you chose (clay or hard paper or piece of wood or cork sheet or any suitable element to help the teamwork), make solids of required sizes keeping the sides equal to $a$, b and c. For example,

1 solid of size $a \times a \times a$ for the term $a^3$

3 solids of size $a \times a \times b$ for the term $3a^2b$

6 solids of size $a \times b \times c$ for the term $6abc$ etc.

Now can you tell in total how many solids are required be made? You can calculate that in total there will be 27 solids. Divide the making of 27 solids among the teammates. Now place the solids together in such a way that they form a cube. You will be amused to obsever that the length of each side of that cube is equal to the length of the stick your teacher brought in class. If it does not happen, there may be some error in the making of the solids. In that case observe the solids carefully and correct any error made.

## Result of the project

- Write the length of the sides and the volume of the cube using variables.
- Find the volume of all 27 solids from their sizes and write the sum of the volumes in terms of the variables.
- As the volume of the large cube is equal to the sum of volume of the 27 solids, we can find the formula from this relation.

## Individual Task

Using algebraic rules, find out the following formulae using the formula for cube of trinomial expressions.

a) $(a + b - c)^3 = a^3 + b^3 - c^3 + 3a^2b + 3ab^2 - 3b^2c + 3bc^2 - 3a^2c + 3ac^2 - 6abc$

b) $(a - b + c)^3 = a^3 - b^3 + c^3 - 3a^2b + 3ab^2 + 3b^2c - 3bc^2 + 3a^2c + 3ac^2 - 6abc$

c) $(a - b - c)^3 = a^3 - b^3 - c^3 - 3a^2b + 3ab^2 - 3b^2c - 3bc^2 - 3a^2c + 3ac^2 + 6abc$

d) $a^3 + b^3 + c^3 - 3abc = (a + b + c)^3 - 3(a + b + c)(ab + bc + ca)$

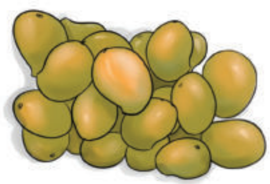
## Factor

You are already familiar with the word 'factor'. You know that factor of a number is another number such that it divides the first number without any remainder. For example – 5 is a factor of 10, because dividing 10 by 5 leaves no remainder. Similarly, 2 is also a factor of 10. In this way there can be more than one factors of a number. Again, 1 is a factor of any number because you can divide any number by 1. A number is also a factor of itself. So, for any number, there are at least two factors: 1 and the number itself. Numbers having only 2 factors are called prime numbers. We can get the idea of factors of algebraic expressions using the idea of factors of numbers. A factor of an algebraic expression is another algebraic expression such that it divides the first expression without any remainder. For example, the expression $x - 1$ is a factor of the expression $x^2 - 1$, because we can divide $x^2 - 1$ by $x - 1$ without any remainder. Similarly, $x + 1$ is also a factor of $x^2 - 1$. In this way one expression can have more than one factors. Like numbers, any algebraic expression can be divided by 1, so 1 is a factor of any algebraic expression. Again, any algebraic expression is a factor to itself. So every algebraic expression has at least two factors. In this part we will discuss about the factors of cubes of binomial and trinomial expressions and their uses.

## Factor of Cubic of a Binomial Expression

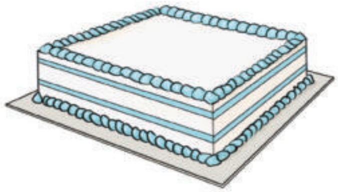
### Factor of numbers (distribution of mangoes)

Afsana wants to equally divide 20 mangoes of their tree among their relatives. Among how many people Afsana can divide the mangoes? You may answer 2 people, because giving 10 to each person will divide the 20 mangoes in two equal parts, that is, 20 is divisible by 10. Here 10 is a factor of 20. Similar numbers by which 20 is divisible are the factors of 20. Even if Afsana wants, she cannot equally distribute by giving 6 mangoes to each person, because 6 is not a factor of 20. Now can you tell which numbers are factors of 20? List the factors in the space below.

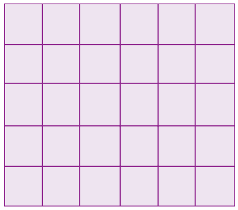


If we want to express this concept using algebraic expressions, we can say that $x$ is divisible by $y$ if $y$ is a factor of $x$.

## Factoring a rectangle (let's cut a cake)



Suppose, you want to divide a rectangular cake into equal pieces. The size of the cake is 12 × 10 sq. inches. How many equal pieces can be made in whole sq. inches? Think for a little while. We are helping you to some extent. We can divide into pieces of 2 × 2 and there will



be 30 equal parts. If you do not believe, take a paper of size 12 × 10. Now put a mark after every 2 inches along the length and the width. Count how many equal pieces are made. Of course, there are in total 30 pieces. As we could divide into pieces of 2 × 2, this is a factor of 12 × 10.

Now think in how many ways the cake of size 12 × 10 sq. inches can be divided. Note down all the sizes and the number of pieces for each size. If needed, verify by marking a paper.

Notice that, there is no piece of size 3×3 in the sizes you found out. Hence 3 × 3 is not a factor of 12 × 10.

Did you find any relation of the sizes of pieces with the size 12 × 10 of the cake. If yes, note that down in the space below.

If you find find any relation of the sizes of pieces with the size 12 × 10 of the cake in some other way, note that down in the space below.

## Factor of cube (Arrange the egg cartons)

Jamal uncle has a shop where he sells eggs. He has a $3' \times 3' \times 3'$ ($3' = 3$ feet) carton of cubic shape where he keeps the boxes of eggs arranged. If each box of egg is the size $1' \times 1' \times 6''$ , then Jamal uncle can make his egg carton full with egg boxes. Because $1' \times 1' \times 6''$ is a factor of $3' \times 3' \times 3'$. Notice that, $1'$ is a factor of $3'$ and $6''$ is a factor of $(3 \times 12)'' = 36''$. Can you tell how many egg boxes of size $1' \times 1' \times 6''$ will be required to fill out the whole egg carton of Jamal uncle?

## Individual task

1. If the size of each egg box is $1' \times 1' \times 4''$, how many egg boxes will be required to fill out the whole egg carton of Jamal uncle?

2. If the size of each egg box is $1' \times 1' \times 5''$, can Jamal uncle fill out his whole carton of eggs using these boxes.

   a) If yes, then how many egg boxes will be required to fill out the whole egg carton?

   b) If no, then what is the reason? What part of the egg carton will remain empty?

## Cube of an algebraic expression and factors of a cube

Now we will establish a relation of factors of a cube using algebraic expression by observing patterns of numerical expressions. Suppose, each cube has length of $x$ units. Then the size of the cube is $x \times x \times x$ and the volume is $x^3$. If p, q and r each is a factor of $x$, then the cube can be divided into equal pieces of size $p \times q \times r$. This means pqr will be a factor of $x^3$ and we can divide the cube in total of $\frac{x^3}{pqr}$ equal pieces. Now, if the value of $x$ is 5, in which sizes we can divide the cube and how many pieces will be there for each size. Note down the sizes and number of pieces in the space below.

Now suppose, the length, width and height of a rectangular solid are respectively $x$, $y$ and $z$. Then the size of the solid is $x \times y \times z$. If p,q and r are respectively factors of $x$, $y$ and $z$, then we can divide the solid into equal pieces of size $p \times q \times r$. This means pqr will be a factor of $xyz$ and we can divide the cube in total of $\frac{xyz}{pqr}$ equal pieces.

## Factoring the cube of a binomial expression

If an algebraic expression cannot be divided by any other expression except 1 and itself then we call that a prime expression. Every algebraic expression can be written as a product of prime expressions. Writing an algebraic expression as a product of prime expressions is called factorization and the prime expressions are called prime factors. We can easily factorize the cube of a binomial expression. For example the cube of x + y can be written as

$$(x + y)^3 = (x + y)\ (x + y)\ (x + y)$$

This is factorization of $(x + y)^3$ . Again, the factorization of $x^3 + y^3$ using algebraic formula gives us,

$$x^3 + y^3 = (x + y)\ (x^2 - xy + y^2)$$

## HCF and LCM of cubic expressions

You have already learned about the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of two or more algebraic expressions. The product of all common prime factors (with repetition) of two or more expressions is the HCF of those expressions. On the other hand, the product of all common prime factors (with repetition), the prime factors of two or more expressions (if any, except all algebric expressions) and the other prime factors is the LCM of those expressions. Let's clarify the topic with an example.

**Example:** Calculate the HCF and LCM of $x^3$, $x^2y$, $x^2y^2$

**Solution:** Here,

Prime factors of the first expression, $x^3$, are: $x, x, x$

Prime factors of the second expression, $x^2y$, are: $x, x, y$

Prime factors of the third expression, $x^2y^2$, are: $x, x, y, y$

This means, from the three algebraic expressions, all common prime factors are: $x$, $x$ (two times). Hence,

$$\text{HCF} = x \times x = x^2$$

Again, all common prime factors of the three algebraic expressions are: $x$, $x$ (two times).

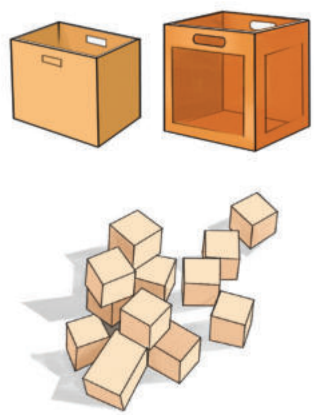
Common prime factor of second and third algebraic expressions is $y$.

Only left out are the prime factor $x$ of the first expression and the prime factor y of the third expression. Hence,

$$\text{LCM} = x \times x \times y \times x \times y = x^3 y^2.$$

## HCF of cubic expressions

Rafid and Rahima are siblings. They have two cubic wooden boxes of length 30 cm and 40 cm respectively. They want to fill up the boxes using same size boxes of mug.



- Which sizes of cube shaped mug boxes (in cm) can be used to fill up the wooden box of 30 cm length?
- Which sizes of cube shaped mug boxes (in cm) can be used to fill up the wooden box of 40 cm length?
- Which sizes of cube shaped mug boxes (in cm) can be used to fill up both wooden boxes?

Fill in the following table 3.4 with your answers.

| Table 3.4 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Cube shaped wooden box** | **Cube shaped box of mug (length in cm)** | | | | | | | |
| 30 cm in length | 1 | | 3 | | 6 | | 15 | |
| 40 cm in length | | | | | | | | |
| Both boxes | | | | | | | | |

What is the highest size of mug boxes that can be used to fill up both boxes? That value is the HCF. Notice that, highest 10 cm length box of mugs can be used to fill up both of the wooden boxes. This means, the HCF of $30^3$ and $40^3$ is $10^3$.

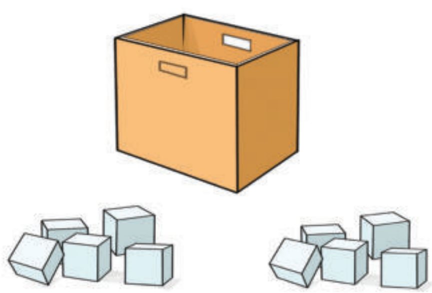
We can determine the HCF of cube of two algebraic expressions by observing the HCF of cube of numerical expressions using relations of algebraic terms. If r is the HCF of $x$ and $y$, then $r^3$ is the HCF of $x^3$ and $y^3$.

## Individual task

What is the length of maximum size cube which can be used to fill up two cube shaped boxes of length 10 units and 6 units?

## LCM of cubic expressions

Now Rafid wants to buy boxes of mug having length 6 cm and Rahima wants to buy boxes of mug having length 8 cm. They want to buy cube shaped wooden boxes to store their mugs. What is the minimum size of wooden box that can be filled up by the boxes of mugs for both? In this case,



how many boxes of mug will Rafid buy?

How many boxes of mug will Rahima buy?

Let's complete the following table.

| Table 3.5 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Cube shaped box of mug** | **Cube shaped wooden box (length in cm)** | | | | | | | |
| 6 cm in length | 6 | | 18 | | 30 | | | |
| 8 cm in length | | 16 | | 32 | | | | |
| Both boxes | | | | | | | | |

What is the lowest size of wooden box that can be filled up by both sizes of mug boxes? That value is the LCM. Notice that, lowest 24 cm length wooden box can be filled up by both sizes of mug boxes. .

We can determine the LCM of cube of two algebraic expressions by observing the LCM of cube of numerical expressions using relations of algebraic terms. If r is the LCM of $x$ and $y$, then $r^3$ is the LCM of $x^3$ and $y^3$.

## Individual task

Can you tell what is the LCM of 10 and 6? If your answer is correct, the cube of your determined LCM is the LCM of cubes of 10 and 6.

## Exercise

1. Which of the following is not a binomial expression? Explain the logic behind your answer.

   a) $xy + 3x$　　b) $xy$　　c) $x + y - 1$

   d) $x^2 - 2x + 1$　　e) $y^2$

2. From the binomial expressions below, separate the expressions with one variable and two variables.

   a) $x + 1$　　b) $3x + 5$　　c) $x - 3$

   d) $5x - 2$　　e) $2x + 3y$　　f) $x^2 + 1$

   g) $x^2 - y$　　h) $x^2 + y^2$

3. From the algebraic expressions below, determine the trinomial expressions of one, two and three variables respectively.

   a) $x + y + 3$　　b) $x^2 + 3x + 5$　　c) $xy + z - 3$　　d) $5x + y^2 - 2$

   e) $2x + 3y - z$　　f) $y^2 - y + 1$　　g) $x^2 - yz + 2$　　h) $x^2 + y^2 - y$

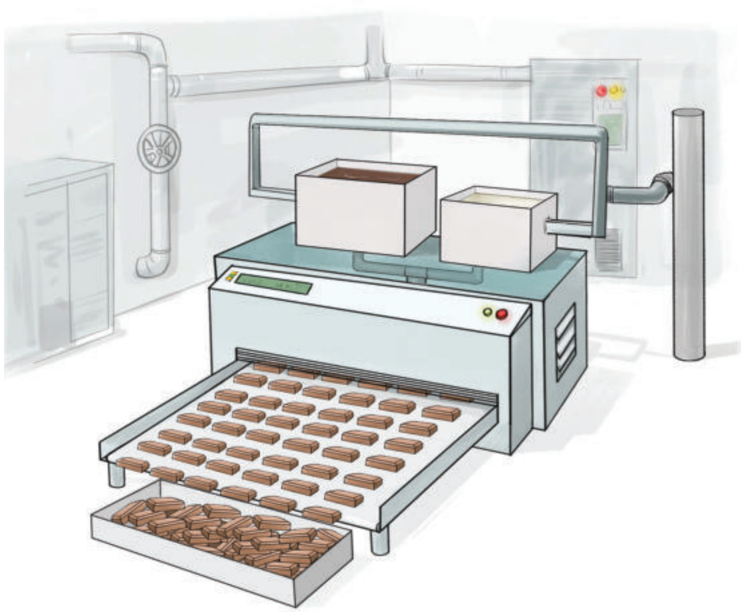
4. Calculate the cubes of following trinomial expressions.

   a) $x + y + 3$　　b) $2x + 3y - z$　　c) $x^2 + 3x + 5$　　d) $xy + z - 3$

5. Factorize the expressions using algebraic rules:

   a) $x^3 + 1$　　b) $x^3 - 1$　　c) $x^6 - 729$　　d) $x^3 + 3x^2 + 3x + 9$

6. In a chocolate factory, raw materials are kept in two cubic shaped containers of length 2 ft and 3 ft respectively to their full capacity.



   a) If no raw material is lost/damaged, how many chocolates of size 1″ × 1″ × 2″ can be produced by taking all the raw materials of the two containers.

   b) If no raw material is lost/damaged, how many chocolates of size 5″ × 7″ × 1″ can be produced by taking all the raw materials of the two containers.

   c) If 1440 chocolate bars of size 5″ × 7″× 1″ are produced using the raw materials of the two containers, how much of the raw materials are wasted?

7. Suppose Lata's father has a fish farm. There is a pond in the farm which is 50 meters long, 40 meters wide and 5 meters deep. Keeping the volume fixed, if we want to reduce the depth by 3 meters, how long the length is needed to be increased?

# Let us build our future with small savings

## You can learn from this experience

- Simple Interest
- Compound Interest
- Profit and loss

# Let us build our future with small savings

Our students are encouraged to do school banking with small savings for a better future. Students can get regular profit by depositing small amount of money. For keeping proper account of savings of different periods and determining profit through investing savings it is essential to know mathematical calculation. In this experience we shall learn to keep accounts of savings and the method of calculating profit/ interest.?

Shanta a student of Grade VIII often gets money from her mother, father and near relatives. Shanta has decided that she will save money and she went to a bank near her residence with her mother to open a Bank account. Shanta said the bank official that she wants to open a bank account and also added that the amount she wants to deposit is 200taka per month. The officer informed that she has to submit necessary documents and 100taka as initial deposit in order to open a bank account . In addition she needs to deposit 200 tk. as monthly instalment. Then she has to deposit tk. 100 regularly per month. Simple interest will be added to this deposit at the rate of 7%. The rate of this interest may change. Shanta opened a bank account following the rules and procedures of the bank.

Do you know what is instalment and what is 7% simple interest ?

The money you have to deposit after a particular period of time is instalment.7% interest means if you deposit tk.100 for one year, the bank will pay you tk. 7 interest.

Can you say,

1. What is the total deposit of Shanta in the first month?
2. What will be the total deposit of Shanta after the end of 2nd month ?
3. What will be the total deposit after 3$^{rd}$ month?

Write down your answer here

## Individual Task

Shanta has prepared the following table in her note book to keep an account. Now fill the table with the total deposit of one year.

**Table 4.1**

| Number of instalment | Amount (in tk. ) | Initial deposit | Cumulative total deposit (taka) |
|---|---|---|---|
| 0 | 0 | 100 | 100 |
| 1 | 200 | - | 300 |
| 2 | 200 | - | 500 |
| 3 | 200 | - | 700 |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |

## Pair work

Observe carefully the table above and find out the relation between number of instalments and total deposit. Write down the relation below.

Match your observation with that of your classmates. If it does not match, it should be considered that anyone of you has done mistake. Discuss with your friend and make necessary correction with the help of the teacher.

[ Total deposit = number of instalment x amount of instalment + initial deposit]

Can you express this relation in a mathematical equation? If we consider $x$ as number of instalment, $m$ as amount of instalment, $c$ as initial deposit, $y$ as total deposit.

**$y = mx + c$ [ Total deposit = amount of instalment $x$ number of instalment + initial deposit]**

Using this equation you can find Shanta's total deposit of any month.

For example, the total amount of deposit on $14^{th}$ month will be,

$$y = mx + c = 200 \times 14 + 100 = 2900 \text{ taka}$$

Using this equation, find and write what will be Shanta's total savings after $16^{th}$ and $23^{rd}$ month

| Number of instalment | Amount of total deposit |
|---|---|
| 16 | |
| 23 | |

## Simple Interest

If you open a bank account, you can get profit or interest at a particular rate. Since Shanta saves regularly, she will receive interest from the bank. Interest can be of two types. Simple interest and Compound Interest. We will calculate simple interest and compound interest. We shall check where she can get more interest.

We have done the calculation of Shanta's deposit. Now we shall do calculation of simple interest that she will get. First we shall learn some rules of interest. Then we will do the calculation of Shanta's interest.

The interest given only on the initial deposit is called Simple interest. For example, if someone deposits 100 tk. in the bank and the bank gives simple interest at the rate of 7% after one year, his interest will be 7 taka, after two years his interest will be 14 tk. and after three years his interest will be 21 tk. . That is , he will get simple interest for every year.

Generally, the bank funds pay the same amount of interest on the amount deposited money. The amount of interest may vary from bank to bank . If a bank fix 7% interest, it means if you deposit 100 tk. a year, they will give 7 tk. profit. Some examples are given below.

## Example 1

What will be 6 year profit of 3000 tk. at the rate of 7% interest?

## Solution

Interest of 100 tk. for 1 year = 7 tk.

$\therefore$ Interest of 1 tk. for 1 year = 7/100 tk.

$\therefore$ Interest of 3000 tk. for 6 years = $\frac{7}{100} \times 3000 \times 6$ tk.

= 1260 tk

Here,

Rate of profit = $\frac{7}{100}$

Principal amount = 3000 tk.

Duration = 6 years

## Pair work

Observe the above solution. Do you find any relation between the rate of interest, principal and duration? Write down after discussing with your partner.

## Algebraic Formula of Simple interest

In the example above we have seen , Rate of Interest = 7% = $\frac{7}{100}$

Principal = 3000 tk.

Time = 6 years

And 6 years interest of 3000 tk = $\frac{7}{100}$ × 3000 × 6 tk.

**Total interest = Rate of interest × Principal × Duration**

If we consider, rate of interest r, Principal P, Duration n, and Total profit I, then you can write down the above relation

$$I = Prn$$

We can also determine some more relations following the above mentioned formula.

$$\text{Rate of interest } r = \frac{1}{Pn}$$

$$\text{Principal } P = \frac{1}{rn} =$$

$$\text{Time } n = \frac{1}{Pr}$$

Using the above Formula some solutions are done below :

### Example 2

What is the interest of 15000 tk. for 3 years at the rate of 12%?

### Solution

From the Formula of Interest, we know that I = Prn

$\therefore$ Interest I = $\frac{12}{100}$ × 1500 × 3 tk.

= 5400 tk.

Here,

Rate of interest,

r = 12% = $\frac{12}{100}$

Principal, P = 1500 tk.

Time, n = 3 years

## Example 3

Rina has received 2200 tk. interest from Bank after 5 years at the rate of 11% simple interest. How much Rina has diposited?

From the Formula of Interest we know that , I = Prn

**Solution: (Do yourself)**

Here,

Rate of interest, $r = 11\% = \frac{11}{100}$

Interest, I = 2200 tk

Time , n = 5 years

## Example 4

If the interest on 4200 tk. for 4 years is 2100 taka, what is the rate of interest?

**Solution: (Do yourself)**

Here,

Principal, P =

Interest, I =

Time , n =

## Example 5

How long will it take to get 12100 tk from principal 44000 tk?

**Solution: (Do yourself)**

Here,

From the example above, it is quite easy to calculate simple interest. But How much simple interest can be earned in one year based on Shanta's savings?

Observe that Shanta's first instalment 200 tk. is deposited in the bank for one year or 12 months. But the 200 tk. of second instalment was deposited in the bank for 11 months. In this way, the amount of next instalments was in the bank for a shorter period of time. If the rate of simple interest is 7% will she earn 7% interest on the total deposit?

Write your answer with explanation in the box below.

## Pair Work

Shanta's account of one year simple interest is partially given below. Fill in the blank space of the table

| Table 4.2 | | | | |
|---|---|---|---|---|
| Serial of the month | Deposited amount (Principal) | Rate of interest r = 7% | Duration n Month | Interest against Principal I = Prn |
| 1 | 200 | $\frac{7}{100}$ | 12 months | Interest = 200 × $\frac{7}{100}$ × $\frac{12}{12}$ = 14.00tk |
| 2 | 200 | $\frac{7}{100}$ | 11 months or $\frac{11}{12}$ years | Interest = 200 × $\frac{7}{100}$ × $\frac{11}{12}$ = 12.83tk |
| 3 | 200 | $\frac{7}{100}$ | 10 months or $\frac{10}{12}$ years | Interest = 200 × $\frac{7}{100}$ × $\frac{10}{12}$ =... ... ... tk |
| 4 | 200 | $\frac{7}{100}$ | **9months or……years** | Interest = 200 × $\frac{7}{100}$ × — = |
| 5 | 200 | $\frac{7}{100}$ | | |
| 6 | 200 | | | |
| 7 | 200 | | | |
| 8 | 200 | | | |
| 9 | 200 | | | |
| 10 | 200 | | | |
| 11 | 200 | | | |
| 12 | 200 | | | |
| Total interest | | | | … … … … |

You can find the total interest to be earned by filling in the blank space. Match/ crosscheck it with your partner and do the necessary correction showing it to your teacher.

## Individual Work

Sumi's mother deposited 1500 tk. per month in her savings account in a bank. She deposited 4 months regularly. However due to special needs, she withdrew all the money before ending the fifth month. If simple interest is 10%, then how much money did she get from the bank?

## Compound Interest

In simple interest we have seen whatever might be the duration of savings, interest is calculated on the initial capital. But to determine compound interest new capital is determined by adding the previous interest.The depositor receives the interest at the end of every year. And on this new capital, the capital of next period is determined. At the end of every period, adding interest with the capital new capital is calculated. The interest a depositor earns on that new capital is called Compound interest. Compound interest is expressed with the sign 'C'. At the end of a particular period the new capital one gets is called Compound capital. It is expressed with the sign 'A'. The starting capital is known as Initial capital. Initial capital is expressed with the sign 'P'. The method of calculating Compound interest is discussed from the example given below.

## Example 6

What is the capital compound and capital interest of/on 3000 tk for 3 years at the rate of 6% interest ?

**Solution:**

1 year interest of 100 tk. = 6 tk

$\therefore$ 1tk.interest of 1 year = $\frac{6}{100}$ tk

$\therefore$ 1 year interest of 3000tk.= $3000 \times \frac{6}{100}$

$= 180$ tk

$\therefore$ Compound in interest after 1 year = initial capital + interest

$= (3000 + 180)$tk

$= 3000 + (3000 \times \frac{6}{100})$ tk

$= 3000\ (1 + \frac{6}{100})$ tk

Since we have got before,

$180 = 3000 \times \frac{6}{100}$

$\therefore$ Equation of compound interest after first year,

$A = P \times (1 + r) \ldots. (1)$

**Now in the second year ,**

1 year interest of 3180 = $3180 \times \frac{6}{100}$ tk

$\therefore$ Compound interest after 2 years = $3180 + (3180 \times \frac{6}{100})$tk

$= 3180 \times \left(1 + \frac{6}{100}\right)$tk

$= \left\{3000 \times \left(1+ \frac{6}{100}\right)\right\} \times \left(1+ \frac{6}{100}\right)$ tk

$= 3000 \times \left(1 + \frac{6}{100}\right)^2$ tk .......(2)

$= 3370.80$ tk

Initial Principal,

P = 3000tk

Rate of interest,

r = 6/100

Compound interest, A

Compound capital

=previous capital + interest

Since earlier we got,

$3180 = 3000 \times \left(1 + \frac{6}{100}\right)$

$\therefore$ From part (2) we can write that, the equation of compound interest after second year

$$A = P \times (1 + r)^2 \quad \dots \quad (3)$$

Initial capital, P = 3000 tk

Rate of interest, $r = \frac{6}{100}$

Compound capital = A

In the same way in third year ,

1 year interest of 3370.80 = $3370.80 \times \frac{6}{100}$ tk

$\therefore$ Compound interest after 3 years

$= 3370.80 + (3370.80 \times \frac{6}{100})$ tk

$= \left(3370.80 \times \left(1+ \frac{6}{100}\right)\right)$ tk

$= \left\{3000 \times \left(1+ \frac{6}{100}\right)^2\right\} \times \left(1 + \frac{6}{100}\right)$ tk

$= 3000 \times \left(1 + \frac{6}{100}\right)^3$ tk ......(4)

$= 3573.05$ tk

So the compound capital after three years 3573.05 tk

And interest is 3573.05 – 3000 = 573.05 tk

$\therefore$ From part (4) we can write that, the equation of compound interest after third year

$$A = P \times (1 + r)^3 \dots\dots.(5)$$

Examine carefully the three patterns of calculating Compound interest, equation no (1), (3), (5) of first, second, and third year. Do you find any relation between A, P, r and n? Write your answer in the box below.

You have certainly observed that all the three equations look alike. There is change only in the value of index and with the value of index there is a relation with n. After observing the equations, can you calculate compound capital of $5^{th}$ year according to example -6 ?

## Formula of Compound capital and Compound interest

After finishing the above task you will see that if the Initial Capital P, Rate of interest r, time n, compound capital A and interest is C then,

Compound capital, $A = P(1 + r)n$

And interest , $C = A - P$

$= P(1 + r)n - P$ [ Since, $A = P(1 + r)n$ ]

$= P[(1 + r)n - 1]$

# Calculating Compound capital

## Example 7

How much will the compound capital on/of 20000 tk be in five year at the rate of 7% Interest?

**Solution:** Do yourself the remaining part.

Compound capital, $A = P(1 + r)^n$

Here,

Initial capital, P = 20000 tk

Rate of interest,

$r = 7\% = \frac{7}{100} = 0.07$

Duration , n = 5

# Calculating Initial Capital

## Example 8

The compound capital will be 20000 tk at the rate of 13 % for 5 years. Find Principal / initial capital

**Solution:** Do yourself the remaining part.

Compound capital, $A = P(1 + r)^n$

$$\therefore P = \frac{A}{(1 + r)^n}$$

Here,

Initial capital , P = 15000 tk

Rate of interest, r = 9% = 0.09

Duration , n = 5

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## Calculating Compound capital

### Example 9

What is the compound interest at the rate of 9% for 7 years?

**Solution:** Do yourself the remaining part.

Compound capital, $C = P[(1 + r)^n - 1]$

Here,

Initial capital , P =

Rate of interest r =

Duration , n =

## Calculating Compound capital during a definite period

Usually interest is given yearly. But the time of compound interest may not be one year. It can be less than one year or fraction of a year. In that case depending on the period the rate of interest is changed. At the same time period should also be changed at a proportionate rate. The idea is explained in the following example.

### Example 10

What will be the compound capital of 2 years of 500 tk for 3 months at the rate of 8%?

Here, to find interest you have to change 8% in yearly proportion of 3 months

3 months = $\frac{3}{12}$ or $\frac{1}{4}$ year

Number of interest to be earned in 1 year $12 \div 3 = 4$ times

Therefore, the number of interest to be earned in 2 years $4 \times 2 = 8$ times, That is, $n = 8$

The rate of compound interest in 3 months $r = \frac{1}{4} \times 8\% = 2\% = 0.02$

Therefore Compound interest will be, $A = P(1 + r)^n$

$= 500 + (1 + 0.02)^8$

$= 500 \times 1.176$

$= 585.83$ tk (approximate value)

## Example 11

What is the interest on 50000 tk in five years if the interest rate is 8% and compounded every 6 month?

**Solution:** In one year interest will be earned two times after every 6 month

$\therefore$ Interest will be earned in 5 years $5 \times 2 = 10$

Compound capital, $C = P[(1 + r)^n - 1)]$

$= 50000[(1 + 0.04)^{10} - 1]$

(Do yourself the remaining part)

Here,

Initial capital, $P = 50000$tk

Rate of interest,

$r = \frac{6}{12} \times 8\% = 0.04$

Time in 5 years, $n = 10$

## Comparison between simple and Compound interest

Previously we calculated total interests of Shanta's one year savings at simple interest rate. Now we shall calculate Shanta's total interest at compound rate and see in which system she gets more interest. Think the bank in which Shanta deposits there monthly interest is given at 7% compound interest every month, then how much will she get at the end of the year?

Since compound interest will be given in every month, Shanta will get compound interest on her first instalment 12 times a year at compound rate. For second instalment she will get compound interest 11 times a year. Interest will be calculated for other instalment in this way.

Here, at the rate of 7% per annum, monthly compound interest rate, $r = \frac{1}{12} \times \frac{7}{100} = \frac{7}{1200} = 0.0058$

The account of Shanta's one year compound interests is given partly in the table below. Fill in the blank space of the table:

**Table 4.3**

| Serial of the month | Deposited amount (Principal) | Rate of interest r = 7% | Duration n Month | Compound Interest, $C = P[(1 + r)^n - 1]$ |
|---|---|---|---|---|
| 1 | Duration | 0.0058 | 12 | Interest = $200[(1 + 0.0058)^{12} - 1] = 14.37$tk |
| 2 | 200 | 0.0058 | 11 | Interest = $200[(1 + 0.0058)^{11} - 1] = 13.14$tk |
| 3 | 200 | | 10 | |
| 4 | 200 | | 9 | |
| 5 | 200 | | 8 | |
| 6 | 200 | | 7 | Interest = $200[(1 + 0.0058)^{7} - 1] = 8.26$tk |
| 7 | 200 | | 6 | |
| 8 | 200 | | 5 | |
| 9 | 200 | | 4 | |
| 10 | 200 | | 3 | Interest = $200[(1 + 0.0058)^{3} - 1] = 3.50$tk |
| 11 | 200 | | 2 | |
| 12 | 200 | | 1 | |
| Total compound capital | | | | … … … … tk. |

What is Shanta's one year interest at compound rate? Write here

What is Shanta's one year interest at simple rate? Write here

So, why will Shanta get more interest? Discuss in pair and write in the box below

## Profit-Loss in Small Business

Small savings can be used to earn income by doing various small businesses. It is important to be able to calculate the profit and loss from the business income properly. Can you tell what is profit and loss? What is the relation of profit and loss with purchase price and sale price? Write your answer below.

What is profit? ____________________

What is loss? ____________________

The amount of money invested in a business is called capital. The capital required to purchase a product is called purchase price of the product and the price in which a product is sold is called the sale price of the product. If the sale price of a product is more than the purchase price, then the sale of the product results in profit and if the selling price of the proucut is lower than the purchase price, the sale of the product results in loss.

Profit = Selling price - Purchase price

Loss = Purchase price - Selling price

The money gained against the purchase price per 100tk.is called profit in percentage. Let's understand the profit and loss issuses form the following examples.

**Example 1:**

Shanta's father is a business man. He bought some goods for his business investing 30 thousand tk.and sold those by 40 thousand tk. at the end of month. What is his profit in percentage?

**Solution:**

Here, investment or capital of Shanta's father = 30000 tk and selling money = 40000tk

Tell me, "Is there any profit or loss in his business?" what is the total amount of profit or loss? Write your answer in the following.

You have surely seen that profit of Shanta's father is 10000 tk. That is,

Profit for 30000 tk. = 10000 tk.

$$\text{Profit for 1 tk} = \frac{10000}{30000} \text{ tk.}$$

$$\text{Profit for 100 tk} = \frac{10000 \times 100}{30000} \text{ tk.}$$

$$= 33\frac{1}{3} \text{ tk.}$$

So, Profit of Shanta's father is $33\frac{1}{3}$ %

## Do yourself

**Problem:** IF a watch seller bought 700 watch at the rate of 350 tk. and sold all of the watches by tk. 2 lacs , then what will be the profit or loss in percentage?

Can you tell, what amount was invested to buy the watches? Write your answer here.

Now, if he sells all watches by tk. 2 lacs, then is it his profit or loss? Give your answer in percentage.

## Individual work

Shanta's mother is a home maker. Along with her household work, she planned to raise goats. For this, she borrowed 5000 tk. from her husband and 10000 tk. from one of her sisters. The condition is that Shanta's mother will get half of the profit after deducting

the expenses from the sale of the goat and the other half of the profit will be shared by the husband and sister in proportion to the amount paid by them. Shanta's mother bought 5 baby goats for 15000 tk. and reared its for a while. She spent 10,000 tk. for its upbringing.

After the goat grew up, she sold it at the village market for Tk 55,000. Who will get how much tk. as profit from the sale of goats?

## Exercise

1. Rais has deposited tk. 350000 for 3 years. If the rate of simple interest is 7%, how much interest will he get after 3 years?
2. Jabin has got tk. 232000 interest from the share of a business with her friend. If the rate of interest is 17% what is her capital in that business?
3. Shimul has received an interest of tk.175000 after investing tk. 80000 for 2 years . What is Shimul's rate of interest?
4. Jony deposited tk. 50000 in the bank. If the rate of interest is 7.5%. How many month would it require to earn an interest of tk. 300000 ?
6. What is the rate of interest if principal-interest of tk.50000 in 7 years is tk.120000?
7. An amount is doubled in 5 years time at a certain rate of interest per annum. In 8 years time the same amount at the same rate will be tk. 26000. Find the amount
8. In 10 years time the interest on tk. 2000 at the rate of 9% per annum will be equal to the interest on tk. 5000 at the rate of 8% for a certain period of time . Find the time in years.
9. Interest on tk. 2500 in 6 years at the rate of per annum will be equal to the amount of interest on tk. 2000 in 8 years at a certain rate of interest per anuum. Find the rate.
10. Tanjila deposited tk.3000 for 5 years and Raihan deposited tk. 20000 for 7 years. If the rate of interest is 8% for both , then who will earn more interest? How much more will s/he earn?
11. Sharif has deposited 70 thousand at 8% interest rate. Jahir gas deposited 50 thousand at 12.5 interest rate. After 6 years who will earn more interest ? How much more will he earn?
12. Calculate 5 years interest of tk .75000 at the rate of 8% interest----

    a) What will be the simple interest ?

    b )What will be the compound interest ?

    c ) What is the difference between simple interest and compound interest ?

d )What is the compound interest of 4 months?

e ) What is the compound interest of 3 months?

13. Jubaer and Rina have deposited tk. 25000 in a bank for 6 years at 7% interest. If Jubaer gets interest at simple rate and Ria gets compound interest rate , who will earn more interest in 6 years? How much will they earn including Principal -interest?

14. Both Ahsan and Tahsina deposited tk. 20000 for 5 years at 11% interest rate for 5 years. If Ahsan earns interest half yearly /per 6 month and Tahsina earns compound interest quarterly/ per 4 months, then who will be more benefitted ? After 5 year what will be the principal of each of them?

15. An individual has taken a loan of tk. 50000 at the rate of monthly 11% compound interest from a Credit organization. If that person pays his loan tk. 12000 monthly then,

a ) How much loan will remain after 1 month?

b ) How much loan will remain after 2 month?

c) How much loan will remain after 3 month?

16. Karim has deposited tk. 50000 for 5 years at compound interest of 9% and Mariam has deposited tk. 80000 for 5 years at compound interest of 7%. Who will earn more from the Bank , How much more will be?

17. What is the profit or loss if Tahsina buys 8 chickens at the rate of Tk 350 and sells them for a total of Tk 2500? What was the capital of Tahsina?

18. A fish farmer released fish worth Tk.5000 in his pond. He spent Tk. 60,000 on fish feed and Tk. 25,000 on fish farm workers. What is the capital of the fish farmer? If he sells his pond fish for 200000 taka, how much profit will he make?

19. A farmer took 20 kg of rice, 5 kg of flour and 1 kg of dal from a shop in exchange of 40 kg of paddy. If the price of one kg of paddy is Tk.12, the price of one kg of rice is Tk.16, the price of one kg of flour is Tk.18 and the price of one kg of pulse is Tk.28, then how much profit or loss did the farmer make?

20. A fruit seller bought 120 hundred lychees for Tk.15000. 600 lychees were lost during the journey. How much rate (per hundred) will he sell the remaining lychees so that his total profit will be tk. 2000?

21. If a bicycle is bought for Tk.5000 and sold at a profit of 12%, how much will be the total profit? What is the selling price of the bicycle?

22. A merchant sold his goods at a loss of 5%. If he could have sold 1230 tk. more, he would have made a profit of 5%. What is the purchase price of the merchant's goods?

23. A manufacturer, wholesaler and retailer, all of them sell a product at a profit of 5%. What is the cost incurred if a customer buys the product from the retailer for tk.1050?

# Triangles and Quadrilaterals in outlines of lands

## You can learn from this experience

- Regular and irregular shapes
- Properties of a right angled triangle
- Pythagoras' theorem
- Use of compass in measurement
- Application of ratios in triangles
- Properties and construction of quadrilaterals
- Exercises of Measuring the outline of school

Academic year 2024

# Triangles and Quadrilaterals in outlines of lands

Let's notice the shapes of the playgrounds, rice fields or house gardens around us. If you observe carefully, you will notice that there are lands of various shapes around us. Consider, some land is parallelogram or trapezium shaped. Can you measure the area? How would you measure the area of a trapezium shaped land, write down in the box below.

Now notice the figure below. We can see the shapes of various lands in a single area. Observe what type of shapes you notice and draw them in your notebook. Think for a while and try to answer how to measure these various shaped lands. Discuss with your classmates, if needed with your teacher and write your opinion in the box below.

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We can easily measure trapezium, rhombus, triangle, or circular shaped lands. In this case, sometimes we can use grids and sometimes we can use formula. But what to do when we face some shapes as we see in the figure? Participating in the discussions and various works, we will learn to identify these shapes easily and measure them in several ways.

## How is the shape of our school's land?

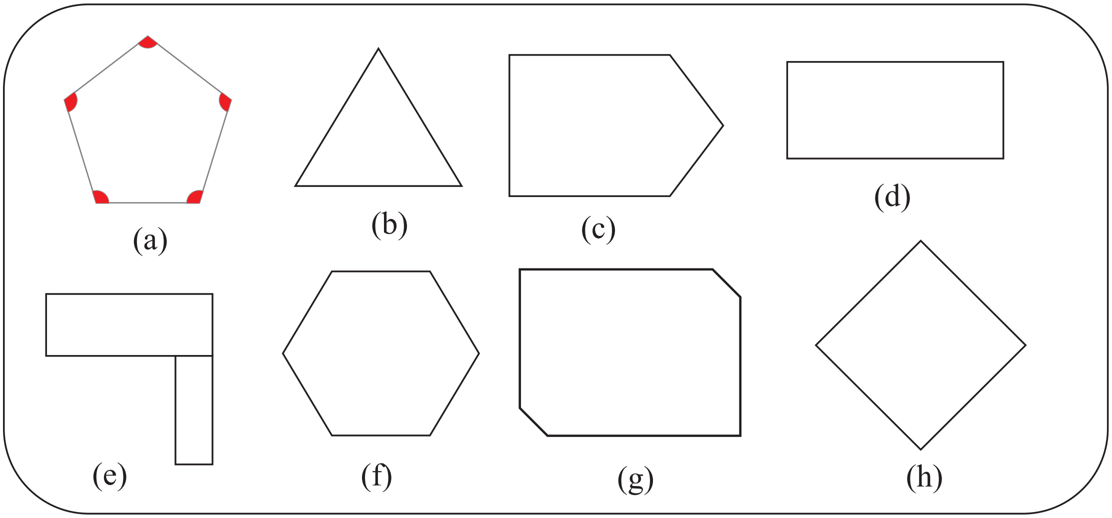
Now we want to give you a task. Get an idea about the shape of your school land area and measure it. Observe the surroundings of your school carefully. Draw an outline of the land area depending on your observations and draw that in the following box.

Let's draw the outline of our school's land

You have drawn the outline of your school land. A very important step for that is to identify various shapes. Let's do a task of identifying some shapes.

## Individual Task

In the box below, various shapes are given. Do you see any similarities by measuring the sides and angles? Identify the shapes which are similar/ have same properties.




Write the reasons for which you identified the similar shapes.

For the individual task,

You found some shapes which have all sides and all angles equal. Those

Again, you found some shapes where all sides or all angles are not equal.

Those are

For a shape, if all sides and all angles are equal then we identify that as a regular shape. Again, if any of the sides or angles is unequal to any other side or angle then that is identified as an irregular shape.

For measurement, identifying regular and irregular shapes is an important step. Now notice, if the outline of your school land is regular or irregular? To measure an irregular shape, we can apply various ideas of triangles and quadrilaterals. Already you have done it for measuring the area of trapeziums. In this part of text, you will know about properties of triangles and quadrilateral in more details and apply those ideas to measure the outline of your school land.

Let's review your previous knowledge about triangles. In your previous classes, you learned in details about acute angle, obtuse angle and right angled trianlges. Fill in the blanks of the quiz in the following table and find out properties of right angles triangles.

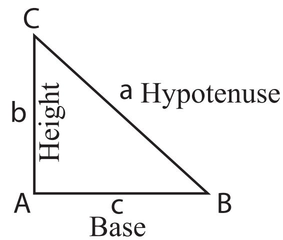



Figure 5.2

## Quiz

- One angle of a right-angled triangle is a _______________.
- What are the names of sides adjacent to the right angle?
- The side which is parallel to the horizontal line is called ___________.
- The side opposite to the right angle is the largest compared to other two sides. This side is called ___________.
- Write the right angle in the box from the given figure. $\boxed{\phantom{xxxxxx}}$
- Determine the area of the triangle using formula ______________.

We can see that we remember the properties of a right-angled triangle well. Let's find out another property through the following task.

## Indiviual task

Draw 5 right angled triangles of various sizes on their notebook. Label the trianlges by numbering them as triangle 1, triangle 2, ... , trinagle 5. Draw squares on the sides of the triangles. Then measure the lengths of sides of the trianlges and write in the table below. Try to write down the calculation of measuring area of squares on the sides and the relation between the areas on your notebook.

**Table 1**

| Triangle no. | Base | Height | Hypotenus | Area of square drawn on the base | Area of square drawn on the height | Area of square drawn on the hypotenus | Relation among the areas |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

Did you find any realtion between the areas? If you measured the length of the sides correctly, you must have observed a realtion. If you could not measure precisely, you can use approximated values to determine the relation. The relation is given in the box. You can match the relation found with your approximated values.

The area of the square drawn on the hypotenus of a right-angled triangle is equal to the sum of the area of squares drawn on other two sides.

If we consider the length of the hypotenus of the right-angled triangle to be c and other two sides to be *a* and b respectively then we can write,

$a^2 + b^2 = c^2$

## Remember this

In the sixth century BC, the Greek philosopher and mathematician Pythagoras determined this special feature of right angled triangle. So it is known as 'Pythagorean theorem'. Though this theorem is named after him, The use of this theorem became remarkable from very ancient times. Its use can be traced back to the Babylonians. It is known, from 800 to 400 BC, many mathematicians of this subcontinent also used this theorem. Afterwards Mathematicians have also interpreted this theorem in different ways.

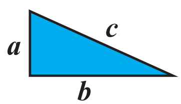
It is assumed that Pythagoras has been born on the island of Samos, near present-day Turkey. His contributions are remarkable to the 'Numerology' and 'geometry related to three dimensions and areas'. Pythagoras was interested in determining the relationships between different numbers which has the direct reflection in Pythagorean Theorem.


Pythagoras

Another interesting phenomenon is **'Pythagorean Trios'** of the Pythagorean Theorem. When the lengths of each sides of right angled triangle are integers then we get Pythagorean Trios. Those three integers which obey the property of right angled triangle is called Pythagorean Trios. For example, (3, 4, 5) and (5, 12, 13) are the two Pythagorean Trios. You can search many more such Pythagorean Trios.

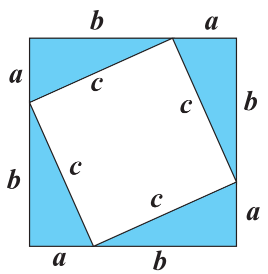
Now let me ask you a question. Is it always possible that the hypotenuse would always be an integer whereas the base and height of the right angled triangle are any integer?

Individual Worksheet– Pythagoras' theorem Proof using papers




We can prove this relation very easily by cutting papers into suitable size. In this case, make four right angled triangle of same size.

Suppose the length of the hypotenus of each triangle is $c$ length of other two sides are $a$ and $b$. Now arrange the 4 triangles into a square shape of length $a + b$ like the figure below.




Here the area of the large square with length $a + b$ is

$= (a + b)^2$

Area of the 4 triangles = $4 \times \frac{1}{2} \times a \times b = 2ab$

Area of the empty space in middle = $c^2$ [Because the empty space makes a square]

**As the area of the large square is equal to the area of the empty space and area of the 4 triangles, you can show that,**

$$a^2 + b^2 = c^2$$

Can it happen that the sum of area of squares on any two sides is equal to the area of square on the other side, but the triangle is not right-angled?

## Individual task:

Take any three lengths $a$, b and c of your choice such that $a^2 \neq b^2 + c^2$ , $b^2 \neq c^2 + a^2$ and $c^2 \neq a^2 + b^2$. Construct a triangle using these three sides. Is any of the angles a right angle? What is the type of the triangle?

If you get $a^2 > b^2 + c^2$ or, $b^2 > c^2 + a^2$ or, $c^2 > a^2 + b^2$ , then the triangle is obtuse angled, otherwise it will be an acute angled triangle.

From this observation we can decide that, if sum of the squares on two sides is not equal to the square on the third side, then the triangle is not right-angled. This means **if sum of the squares on two sides is equal to the square on the third side, then the triangle is right-angled.** This result is called **the opposite theorem of Pythagoras' theorem.**

## Learning various techniques of measuring shapes

Often we need to construct triangles and quadrilaterals on given information. We learnt this construction in previous classes. Again, the following individual task can be easily done using a protractor. But what can you do if you do not have a protractor?

## Individual task

Suppose you have to do the following work to draw the outline of a land or structure.

- Construction of an angle equal to a given angle (∠A).
- Bisect the angle you have drawn.
- Drawing a perpendicular to a line at a fixed point

## Using compass for measurement

a) Suppose you want to draw an angle equal to a given angle (∠A).

- Take any ray PQ. Then place the needle of the pencil compass on A and draw an arc.
- The arc intersects the rays at points B and C. Taking the same radius, draw another arc centering P. The arc intersects the ray at point Q. Now draw another arc centering Q taking radius equal to BC.

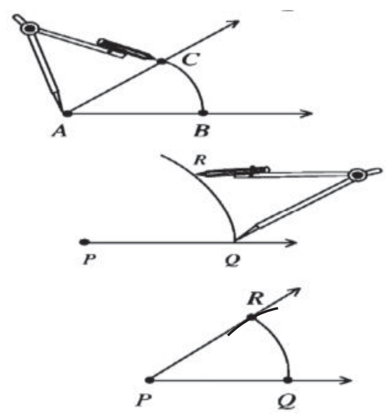
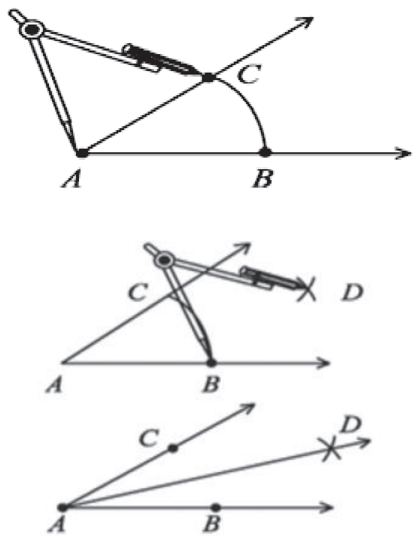



Figure 5.4

- The new arc intersects the first arc at point R. Now join the points P and R and extend the ray away from P.
- Verificarion– Now measure the angles ∠RPQ and ∠BAC to verify if they are equal.

b) Again, suppose you want the bisect an angle, ∠A.




- Draw an arc taking any radius centering the point A. It intersects the two rays of the angle at points B and C.
- Now draw an arc taking radius more than half of BC centering B. Taking the same radius, draw another arc centering the point C.
- Suppose the arcs intersect each other at point D. Join the points A and D. Measure if the angles ∠BAD and ∠CAD are equal.
- Verification– You can also check the equality of two parts by folding papers.

c) Consdier that you want to draw a perpendicular to a line at a fixed point. You can do that using ruler and compass only.

- First consider a line segment AB. Then fix a point C on the line segment.
- You will draw a perpendicular on the point C. Draw an arc centerting C of any radius. Suppose the arc intersects the line segment AB at points E and F. Now draw two arcs of radius of length more than half of EF taking E and F as centers on the same side of AB. Suppose they intersect at the point P. Join the points P and C. Then PC is the required perpendicular on AB.

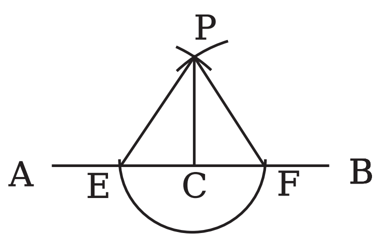



Figure 5.6

- Verification- You can measure the angles on both sides of PC to check if it is perpendicular to AB. But it can also be verified using mathematical logic. In that case, join P, E and P, F to form the $\Delta PEC$ and $\Delta PFC$. According to the construction, in these triangles EC=FC, PE=PF and PC is common to both triangles. This means that the sides of one triangle are equal to sides of another triangle. Hence the triangles are congruent. So $\angle PCE = \angle PCF$. Again $\angle ECF$ = straight angle = two right angles. So, each of the angles is a right angle.

  Hence, PC is perpendicular to AB.

Do you have any idea of doing the above constructions without using compass or protractors? Write down your ideas here.

## Use of ratios in triangles

In this part of experience, you will investigate some important properties regarding ratios in triangles while constructing various shapes and measuring them. You can measure any shape by finding a triangle inside that shape and using the properties of ratios of triangles. Again, you can compare two triangles and decide something during the measurement.

## Pair Work

You already know that, area of a triangle $= \frac{1}{2} \times$ base $\times$ height. Make a team of two students each. Each team sould construct five triangles, all having base 3 cm. Measure the height of each triangle and fill in the table given. Do you get any realtion

**Table 5.2**

| S.No. | Base | Height | Area | Area/ Height |
|---|---|---|---|---|
| 1. | 3 cm | | | |
| 2 | 3 cm | | | |
| 3. | 3 cm | | | |
| 3. | 3 cm | | | |
| 5. | 3 cm | | | |

between the area and the height? Write down your decision below.

Decision of Group Work-

If you calculate the area of the triangles correctly then you can notice that the following statement is true.

If bases of two triangles are equal, then their areas are directly proportional to their heights.

Alternate proof: You can also prove the above statement using the formula of area of triangle.

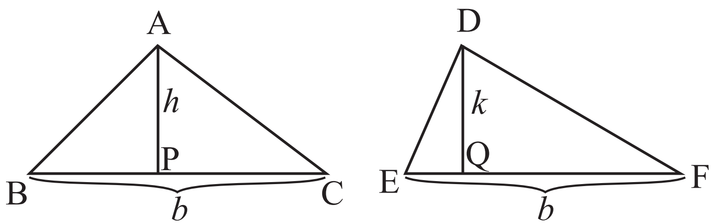



Figure 5.7

If we consider that ΔABC and ΔDEF hasve equal base of length b and their heights are respectively h and k, then we get,

$$\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta DEF} = \frac{\frac{1}{2}\times b\times h}{\frac{1}{2}\times b\times k} = \frac{h}{k} = \frac{\text{Height of }\Delta ABC}{\text{Height of }\Delta DEF}$$

বা, $$\frac{\text{Area of }\Delta ABC}{\text{Height of ABC}} = \frac{\text{Area of }\Delta DEF}{\text{Height of }\Delta DEF}$$

So, we can decide if bases of triangles are equal, then their areas are directly proportional to their heights.

**Individual Worksheet-** Construct five triangles of same height but different bases. Measure the bases and areas and show if heights of triangles are equal, then their areas are directly proportional to their bases. Show an alternative proof for this statement.

Here we obtained the relation of ratios of area of a triangle with its base and height. There are some other properties of triangles from which we can get relation of ratios of triangles. Let's perform the following task.

- Draw a triangle ΔABC.
- Draw a line DE parallel to BC by folding papers or any other method.
- Suppose the parallel line intersects the sides AB and AC at the points D and E respectively. Measure the lengths of the following segments to complete the table.

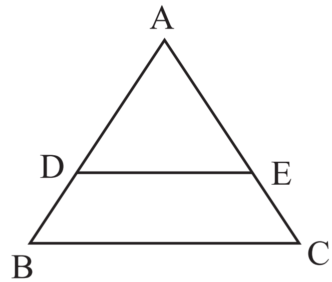


Figure 5.8

| Table 5.3 | | |
|---|---|---|
| **Use same unit of length for all lines** | | **Ratio** |
| AD= | DB= | AD/DB = |
| AE= | CE= | AE/CE = |

Discuss about the results yourselves. Is your decision same as the one below?

In a triangle, a line parallel to any side of a triangle divides the other two sides by the same ratio.

Now let's think about this in a different way. Draw a triangle ΔABC.

Extend the sides AB and AC up to D and E respectively in such a way that DE is parallel to BC. Then make a similar table like above and observe the results. We can write the final decision in the following way.

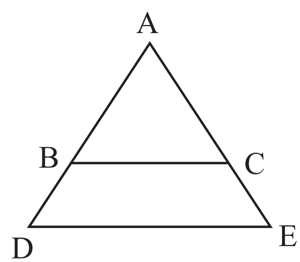


Figure 5.9

In a triangle, a line parallel to any side of a triangle divides the other two sides or their extensions by the same ratio.

## Group Work

Prove that, if a straight line divides two sides of a triangle or their extensions in the same ratio then the line is parallel to the third side.

Using this result, we can easily divide any line segment into a fixed ratio. For example, you want to divide a line segment of 9 cm internally. We can follow the given steps to do that easily.

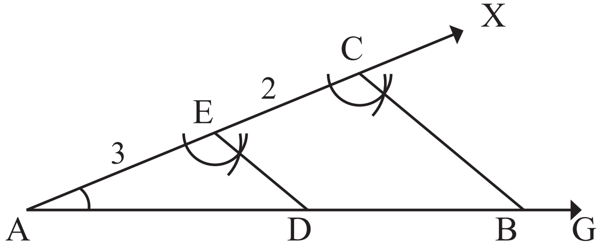



Figure 5.10

- First draw a ray AG. Then mark AB = 9 cm from AG.
- Draw an angle ∠BAX of any measure at the point A.
- Cut off AE = 3 cm from AX and cut off EC = 2 cm from EX.
- Join the points B and C. Now construct a line ED parallel to BC through the point E which intersects AB at D.

Now measure AD and BD to verify $\frac{AD}{BD} = \frac{AE}{EC} = \frac{3}{2}$. As AD + BD = AB = 9 cm and AD : BD=3 : 2, hence a line segment of length 9 cm is divided internally by the ratio 3 : 2.

### Individual task:

Suppose you have a chord of any length. Divide the chord internally by the ratio 5:3.

Let's investigate another property regarding ratios in triangles.

## Group Work

Form teams of three members. Each team draw a triangle ΔABC of any measure. Draw the internal bisector AD of ∠A by folding papers or any other method. Measure the lengths to fill up the following table:

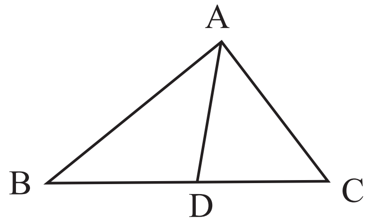



Figure 5.11

| Table 5.4 | | |
|---|---|---|
| Use same unit of length for all lines | | Ratio |
| BD= | DC= | BD/DC= |
| AB= | AC= | AB/AC= |

Discuss the results within your team. Match the decisions of your team with others. Match the final result with the following statement.

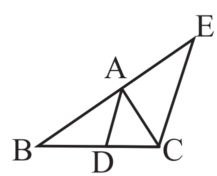
The internal bisector of any angle of a triangle divides the opposite side in the ratio of the sides adjacent to the angle.

## Individual task:

1. In a land of triangular shape ΔABC , a line DE intersects the sides AB and AC in such a way that AB : BD = AC : CE. If the area of the land shaped as ΔDBC is 10 sq. meters, what is the area of ΔBEC?

2. In a land of triangular shape ΔABC , a line DE parallel to BC intersects the sides AB and AC at points D and E respectively. If AE : CE = 3 : 2 and BD = 2 m, what is the length of AB?

3. Suppose the bisector of ∠A intersects the side BC at the point D. Here AD is parallel to CE and BD : DC = 3 : 2. If AE = 10m, determine the length of AB.




Academic year 2024

## Various quadrilaterals around us

While making an outline or measuring the area of lands, ideas on quadrilaterals also helps us with triangles. We have learned various types of quadrilaterals in previous classes. Identify various types of quadrilaterals in the figure below. Measure the shapes provided in the figure for this task.

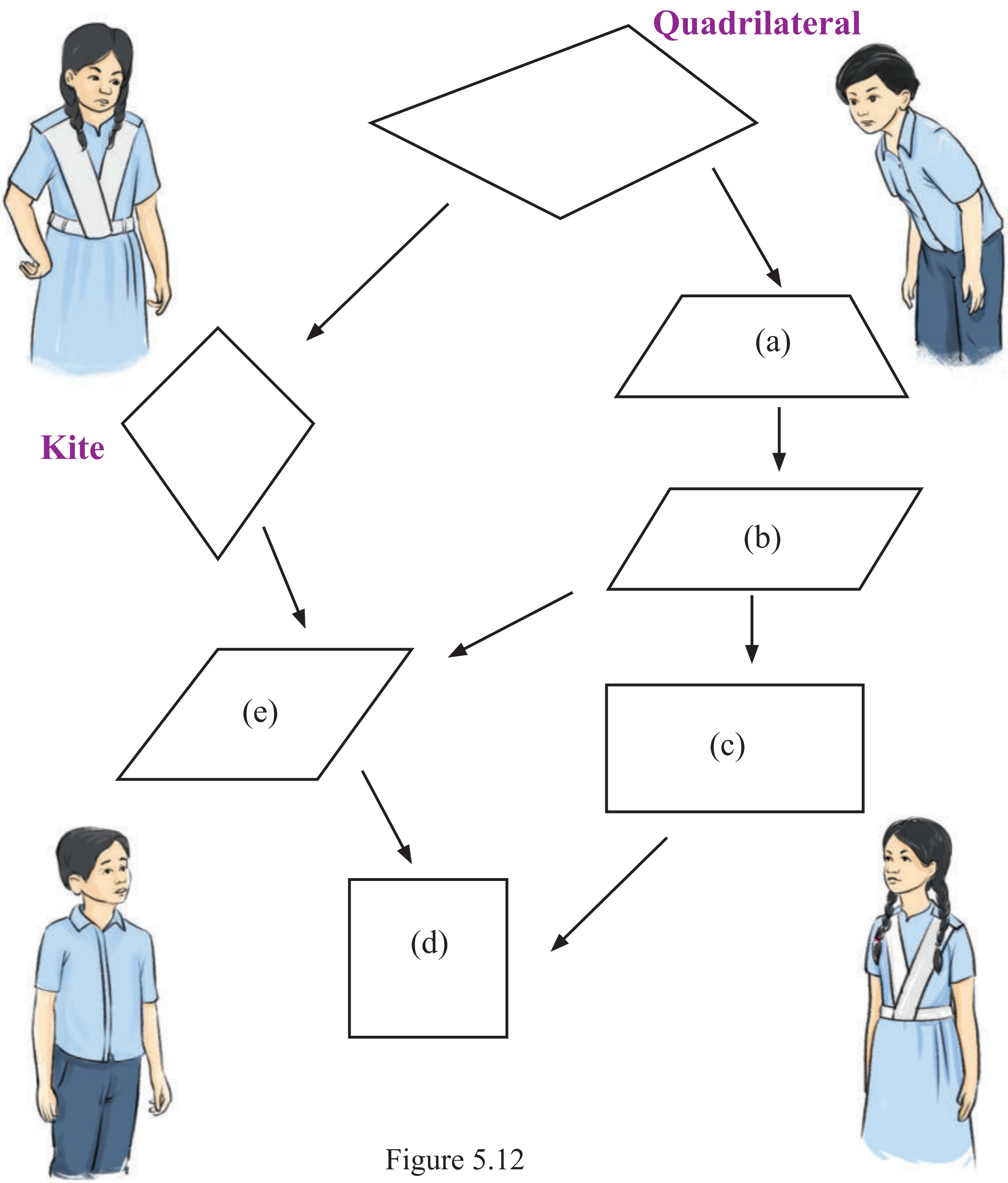



Figure 5.12

Fill the table below writing the names of the shapes. Measure the sides and angles of the shapes for this. Write your reasoning behind your decision.

| Table 5.5 | | |
|---|---|---|
| **Shape** | **Name of the shape** | **Reason behind your decision** |
| (a) | | |
| (b) | | |
| (c) | | |
| (d) | | |
| (e) | | |

You must have noticed that there is a new shape which was not introduced in your previous class. This shape is called a kite. In a kite, two pairs of adjacent sides are equal. Again, if the sides from both pairs are equal, that is, all four sides are equal, then we call that a rhombus. So, each rhombus is also a kite. In the same way you can find relation among other pairs too. By answering the questions from the following table, you can find those relations. Now discuss in pairs to complete the table.

| Table 5.6 | |
|---|---|
| **Question** | **Answer** |
| For which properties of a quadrilateral, we call it a trapezium? | |
| For which properties of a trapezium, we call it a parallelogram? | |
| For which properties of a parallelogram, we call it a rectangle? | |
| For which properties of parallelogram, we call it a rhombus? | |
| Is a square also a rhombus? Give reasons behind your answer. | |
| For which properties of a rectangle, we call it a square? | |
| Is a square also a parallelogram? Give reasons behind your answer. | |
| Is a parallelogram also a trapezium? Give reasons behind your answer. | |

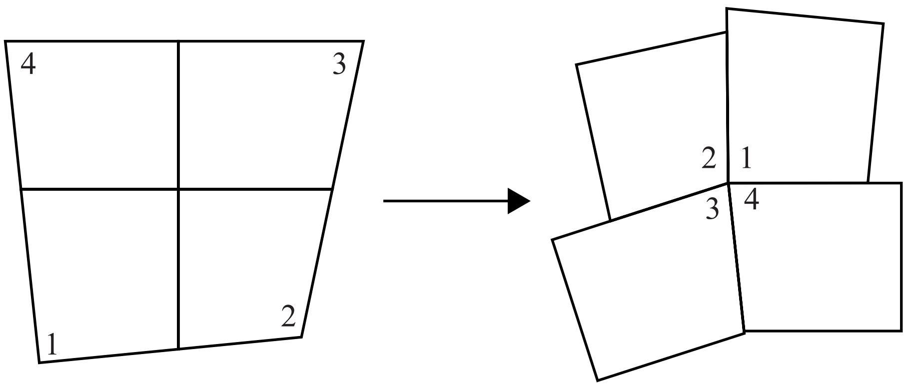
Fill the blank spaces using the given names of shapes.

**rectangle, trapezium, square, parallelogram, rhombus, quadrilateral**

A shaped enclosed by four sides is called a _____________. If a pair of opposite sides of a triangle are parallel, then it is called a _________________. If both pairs of opposite sides are parallel, then it is called a ______________. Again, if one angle of a parallelogram is a right angle, then we obtain a ____________. If adjacent sides of a rectangle are equal, we get a _________. On the other hand, if adjacent sides of a ____________ are equal we get a rhombus and if one angle of a ___________ is a right angle then it is called a square.

## What is the sum of four angles of a quadrilateral?

You have already found out some properties of various quadrilaterals and their relations. Now you will find out the sum of the angles of a quadrilateral. Draw a quadrilateral on your own of any measure. Then cut the figure into four pieces so that the four angles are in four pieces. Arrange the pieces on a same point as the figure bellow.



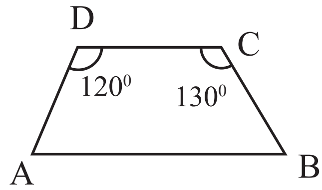

Can you calculate the sum of the angles of the quadrilateral? Did everyone draw quadrilaterals separately? Did everyone get the same result? Check after discussing among yourselves. Is your decision same as the following?

Sum of the angles of a quadrilateral is equal to four right angles or 360°

## Individual task:

1. ABCD is a parallelogram. If ∠A = 60° then ∠B=?

2. ABCD is a trapezium where AB and CD are parallel.




∠A = ? ,  ∠B = ?

## Investigating properties of a parallelogram

The four vertices of a parallelogram can be considered as two pairs of opposite vertices. Let's look for the relations between each pair of angles at opposite vertices.

- Construct a parallelogram ABCD by yourself.
- Divide the parallelogram along the diagonal BD into two triangles △ABD and △BDC.
- Now place the triangle △BDC on the triangle △ABD in such a way that the point C falls on A, the point B falls on D and the point D falls on B. In this case the side BC will fall on DA and CD will fall on AB.

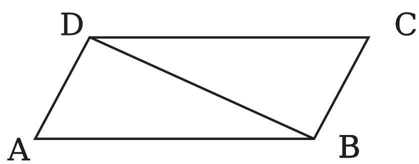


Figure 5.13

- Notice that the triangles coincide with each other. So we can say that ∠A = ∠C, AD = BC, AB = CD.
- Similarly dividing along the diagonal AC we can show that ∠B = ∠D.

Hence the following statement can be made:

**In a parallelogram, the angles at opposite vertices are equal and the opposite sides are also equal.**

Again, consider a parallelogram ABCD whose diagonals are AC and BD.

- Suppose, the diagonals of the parallelogram are intersecting at point O.
- Now place the point C on the point A by folding.
- Then identify the middle point of AC by opening the fold.

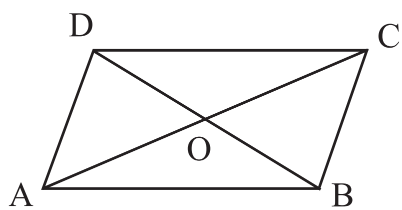


Figure 5.14

- In a similar way by placing B on D by folding and opening the fold identify the middle point of the diagonal BD.
- Match them and see that both the midpoints are same. This point is the same as the intersecting point of the diagonals AC and BD. This means that the diagonals of a parallelogram bisect each other.

Verification –We can verify this decision in another way by measuring the distance from the intersecting points of the diagonals to the vertices. You can verify the result in this way for the parallelogram you constructed.

## Individual Task

Does the same property as parallelograms work for diagonals of rhombus, rectangle and square? Check yourself.

We know, four sides of a rhombus are equal. Construct a rhombus ABCD and join the opposite vertices to form the diagonals AC and BD. Suppose the diagonals intersect each other at O. Measure the following angles and fill in the blanks.

$\angle AOB$ = - - - - degrees, $\angle BOC$ = - - - - degrees,

$\angle COD$ = - - - - degrees, $\angle DOA$ = - - - - degrees.

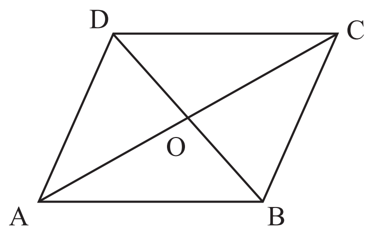


Figure 5.15

$\angle AOB + \angle BOC$ = - - - - degrees = $\angle AOC$ = one straight angle.

$\angle BOC + \angle COD$ = - - - - degrees = $\angle BOD$ = one straight angle.

$\angle COD + \angle DOA$ = - - - - degrees = $\angle COA$ = one straight angle.

$\angle DOA + \angle AOB$ = - - - - degrees = $\angle DOB$ = one straight angle.

The diagonals of a rhombus bisect each other at --------- degrees.

The diagonals of a rhombus bisect each other at right angles.

## Individual Task:

Prove that the diagonals of a square bisect each other at right angles and submit the proof in a worksheet.

Already we know from the tasks that the diagonals of a rectangle bisect each other. But are they equal? Let's learn from the following task.

## Individual task

Fill in the blanks of the table-5.7 using the information from the figure-5.16 and write your decision.

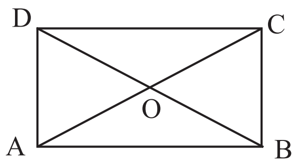



Figure 5.16

**Table 5.7**

| Proposition (Between triangles $\triangle BAD$ and $\triangle CAD$) | Reason |
|---|---|
| $AB = CD$ | |
| $AD = AD$ | |
| $\angle BAD = \angle CDA$ | |
| $\therefore \triangle BAD$ and $\triangle CAD$ are congruent | If for any two triangles, any two sides and the angle between them are equal then the triangles are congruent. |
| $\therefore BD = AC$ | |

Decision: ____________________________________________

## Construction of quadrilaterals

We have learned about various properties of quadrilaterals which will help us measuring shapes and areas. Now we will work on construction of quadrilaterals of various types. In your previous class you studied about constructing square, rhombus, rectangle, and parallelogram by game of four sticks and drawing them on your notebook. All these shapes were quadrilaterals. We can make other types of quadrilaterals using four sticks which cannot be given any special name. So overall all shapes are quadrilaterals. But the question is, can we construct a quadrilateral with four sticks of any length?

You faced this problem while constructing triangles. There you could not form triangles with any three sides. The condition was that sum of any two sides must be greater than the other side. Similarly for quadrilaterals try with sticks of various lengths and write down your decision in the following empty box.

Now discuss with your classmates to check if you made any mistakes. If needed correct your decision.

As for the construction of a triangle we need sum of any two sides to be greater than the other, similarly for the construction of a quadrilateral we need sum of any three sides to be greater than the other side. Otherwise we cannot construct the quadrilateral.

## Group Work

Let's construct some quadrilaterals. For this task you may use geometry boxes.

(a) Construct the quadrilateral ABCD where AB = 3 cm, BC = 4 cm, CD = 5 cm and DA = 6 cm.

Are the quadrilaterals drawn by various teams the same? Of course, no. As you don't know the measure of the angle between two sides, you had to take that on your own. So quadrilateral of each team was different shaped. So we can fix an angle with the four sides and check how the quadrilaterals look.

(b) Construct the quadrilateral ABCD where AB = 3 cm, BC = 4 cm, CD = 5 cm, DA = 4.5 cm and the angle between the sides AB and AD is 60 degrees.

Now notice that, quadrilaterals of all teams are exactly the same. So we can say that, four sides are not enough to construction a specific quadrilateral, you need to fix an angle too. But there is another matter. If you don't join the sides in the order defined, do you get the specific quadrilateral? Are the quadrilaterals same for all teams?

Discuss with your classmates to write your decision below.

**So, to construct a specific quadrilateral with four sides and an angle we need to specify the order of four sides and between which sides the angle would be. Only then, we will get a specific quadrilateral.**

(c) Now we will check if we can construct a specific quadrilateral if we have four sides and one diagonal. Suppose the length of the sides are 4 cm, 4.5 cm, 5 cm, 3.5 cm and the length of the diagonal is 6.5 cm.

## Directions for construction:

- Take a line segment equal to the diagonal from any straight line. Taking center at one endpoint draw arcs taking radii equal to any two sides.
- In this case you can take length of any two sides.
- Similarly, from the other endpoint draw two more arcs taking radii equal to the length of other two sides.

- Join the endpoints of the diagonal from the intersecting points of the arcs to complete the quadrilateral. Now match everyone's quadrilateral to check if everyone has the same quadrilateral.

If the quadrilaterals are not the same, write down the reasons and what conditions can be imposed to make them the same in the box below.

Show this to your teacher and correct them if needed by taking your teacher's advice.

Now notice what are the parts of a quadrilateral. A quadrilateral has four sides, four angles and two diagonals. Out of this ten information, we chose five and constructed a specific quadrilateral. Now we will take five information in various ways and try to construct a specific quadrilateral. Later you can use these properties of a quadrilateral in drawing designs of various shapes.

## Three sides and two diagonals

Suppose, the three sides are $a$ = 3 cm, $b$ = 4 cm, $c$ = 3.5 cm and two diagonals are $d$ = 4 cm, e =5 cm .

Using this information, figure-5.17 is drawn.

In the following table-5.8, the construction is described in jumbled order. Write the description in proper order.

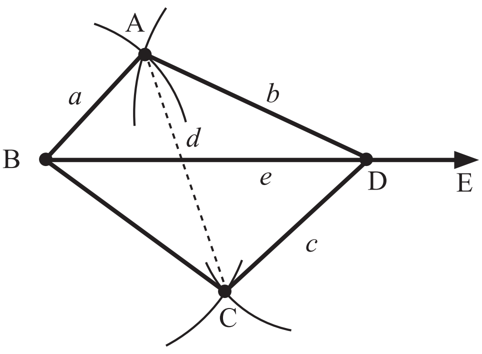



Figure 5.17

| Table 5.8 | |
|---|---|
| Description of construction (jumbled order) | Description of construction (proper order) |
| Take BD = $e$ =5 cm from any ray BE . | |
| Draw an arc centering D taking radius $c$ = 3.5 cm on the opposite side of BD where A is situated. | |
| Suppose the arcs intersect at point A. | |

| | |
|---|---|
| Draw another arc centering D taking radius $b$ = 4 cm on the same side of BD. | |
| Draw an arc centering B taking radius $a$ = 3 cm on any same side of BD. | |
| Suppose the arcs intersect at point C. | |
| Join A and B, A and D, B and C, C and D. | |
| Draw an arc centering A taking radius $d$ = 4 cm on the opposite side of BD where A is situated. | |
| Hence ABCD is our required quadrilateral. | |

Now construct the quadrilateral yourself following the steps and verify your description. Also, match to check if everyone got the same quadrilateral.

## Three sides and the two angles between them

Suppose, lengths of three sides are $a$ = 6 cm, $b$ = 5 cm, $c$ = 4 cm and the angle between $a$ and $b$, $\angle x=80°$, and the angle between b and c, $\angle y=70°$. Construct the quadrilateral.

In the following table-5.9, the construction is described in jumbled order. Write the description in proper order and construct the quadrilateral.

**Table 5.9**

| Description of construction (jumbled order) | Description of construction (proper order) |
|---|---|
| Draw an angle ∠CBF at the point B measuring $\angle x = 80°$. | |
| Draw an angle ∠BCG at the point C measuring $\angle y=70°$. | |
| Cut off $c$ = 4 cm = CD from CG. | |
| Cut off $b$ = 5 cm =BA from BF. | |
| From any ray BE, cut off BC = $a$ = 6 cm . | |
| Hence ABCD is our required quadrilateral. | |
| Join the points A, D. | |

## Two adjacent sides and three angles

Suppose the length of two adjacent sides and three angles are given for a quadrilateral. We need to construct the quadrilateral.

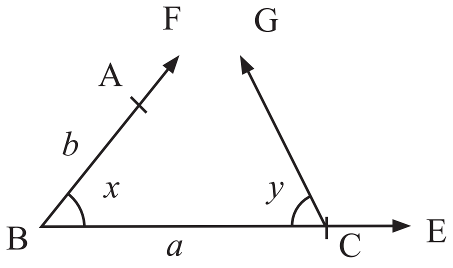



Figure 5.18

**Individual task:** Suppose, two adjacent sides of a quadrilateral are $a$ = 5 cm, $b$ = 6 cm and the angle between them is $\angle x = 70^\circ$, and other two angles adjacent to the sides are respectively $\angle y = 80^\circ$, $\angle z = 100^\circ$. Construct the quadrilateral with description of construction. One partially completed rough figure-5.18 is given.

## Determining the area

In your previous class you learnt about determining areas of various types of triangles and quadrilaterals. Here the formula for various triangles and quadrilaterals are given. Place them accordingly in the table.

$\frac{1}{2}d_1d_2,\quad bh,\quad a^2,\quad ab,\quad dh,\quad \frac{1}{2}bh,\quad \frac{h(a+b)}{2}$

| Table 5.10 | |
|---|---|
| **Shape** | **Area** |
| Rectangle (length $a$ and width b) | |
| Square (length of sides $a$) | |
| Parallelogram (base $b$ and height $h$) | |
| Parallelogram (length of $a$ diagonal is $d$ and the length of the perpendicular on the diagonal drawn from $a$ vertex not on the diagonal is $h$) | |

| | |
|---|---|
| Rhombus (diagonals are $d_1$ and $d_2$ ) | |
| Trapezium (lengths of parallel sides are $a$ and $b$, and their perpendicular distance is h ) | |
| Triangle (base $b$ and height $h$) | |

You know that the area of a triangle = $\frac{1}{2}$ × base × height. This means, to know the area of a triangle you must know about the base and height. Suppose you don't know the height; you just know the length of three sides. Let's see if we can calculate the area of a triangle using the length of three sides.

Suppose we have a triangle ΔABC where BC = $a$, CA = $b$, AB = $c$ and AD is the perpendicular on the base BC. We don't know the length of the perpendicular AD.

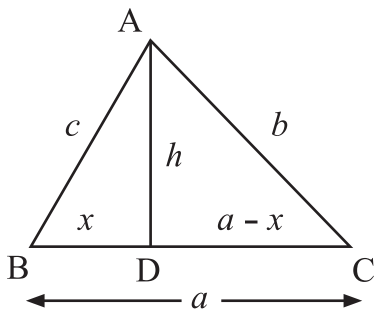



Figure 5.19

Here, we know the base BC = $a$. If we can express the height of the triangle in terms of the lengths of the sides, then we can find the area of the triangle using the formula known to us. The perpendicular AD divides the triangle ΔABC into two right-angled triangles. So, by applying Pythagorus's theorem, we can find the length of AD.

Suppose, AD = $h$ and BD = $x$ , hence CD = $a - x$ .

Use the property of right-angled triangles on ΔABD and ΔACD and complete the box below.

$AB^2 =$

$AC^2 =$

$AB^2 - BD^2 = AC^2 - CD^2$

Or,____________________________________________

_______________________________________________

_______________________________________________

$\therefore x = \frac{c^2+a^2-b^2}{2a}$

and $AD^2 = c^2 - x^2$

$$= c^2 - \left(\frac{c^2+a^2-b^2}{2a}\right)^2$$

$$= \left(c + \frac{c^2+a^2-b^2}{2a}\right)\left(c - \frac{c^2+a^2-b^2}{2a}\right)$$

$$= \frac{2ac+c^2+a^2-b^2}{2a} \cdot \frac{2ac-c^2-a^2+b^2}{2a}$$

$$= \frac{\{(c+a)^2-b^2\}\{b^2-(c-a)^2\}}{4a^2}$$

$$= \frac{(c+a+b)(c+a-b)(b+c-a)(b-c+a)}{4a^2}$$

[Let, $a + b + c = 2s$;

Here "$s$" denotes the semi perimeter of the triangle.]

$$= \frac{2s(2s-2b)(2s-2a)(2s-2c)}{4a^2}$$

$$= \frac{4s(s-a)(s-b)(s-c)}{a^2}$$

$\therefore \boldsymbol{AD = \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2}.a.\frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{s(s-a)(s-b)(s-c)}$

Calculating $s = \dfrac{a+b+c}{2}$, using the above formula we can determine the area of any triangle using the lengths of the sides.

If the triangle is isosceles, then let the sides be $a, a, b$ . So, $s = \frac{a+a+b}{2} = \frac{2a+b}{2}$

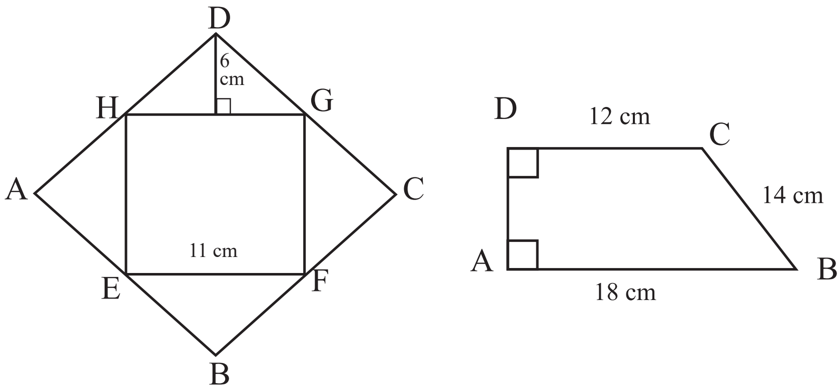
$\therefore s - a = \frac{2a+b}{2} - a = \frac{2a+b-2a}{2} = \frac{b}{2}, \; s - b = \frac{2a+b}{2} - b = \frac{2a+b-2b}{2} = \frac{2a-b}{2}$.

**From this information show that the area of isosceles $\boldsymbol{\Delta ABC = \frac{b}{4}\sqrt{4a^2 - b^2}}$**

Again, if the triangle is equilateral, let the sides be $a, a, a$. So, $s = \frac{a+a+a}{2} = \frac{3a}{2}$
and $s - a = \frac{3a}{2} - a = \frac{3a-2a}{2} = \frac{a}{2}$.

**From this information show that the area of equilateral $\boldsymbol{\Delta ABC = \frac{\sqrt{3}}{4}a^2}$**

Observe the figure: 5.20. Here are two figures. How can we determine the area of them?




The midpoints of the sides of the square ABCD are respectively E,F,G and H.]

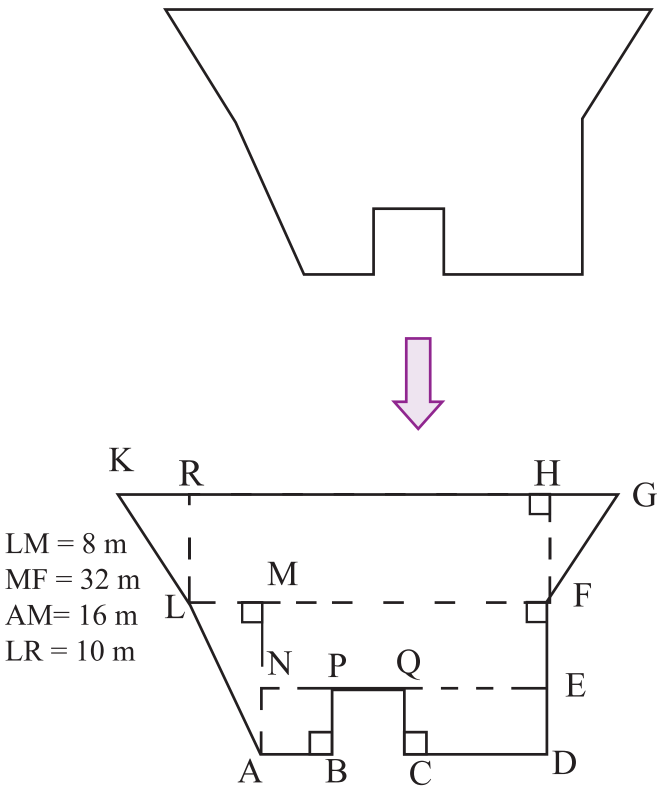
Figure: 5.20

While determining the area in the first figure, notice that we formed a quadrilateral EFGH by joining the midpoints of the sides of the square ABCD. As we have AE = EB = BF = FC = CG = GD = DH = HA and ∠A = ∠B = ∠C = ∠D, there are four isosceles triangles on four sides who are congruent to each other. Determining the area of the square in the middle and adding the area of the triangles around, you can easily find out the area of the whole structure.

Notice the second figure. This is a trapezium. The distance between the parallel sides is not given. You can draw a perpendicular on the line AB from point C and divide the trapezium into a rectangle and a triangle. Here, the base of the triangle will be 18-12 cm=6 cm. Finally using Pythagorus's theorem you can find out the height of the triangle which is also the distance between the parallel sides of the trapezium.

## Individual Work

Triangle and quadrilateral are mrked in the figure 5.21. Find the total area of the land.




AB = 8 m, CD = 16 m, DE = 6m, EF = 8 m, FH = 10 m, KG = 56m

| Area = ________________ Sq. m. |
|---|

Figure 5.21

You have learned about various topics regarding the properties and constructions of triangles and quadrilatearals. Now, take the outline of your school land you prepared earlier and divide it into suitable triangles and quadrilaterals.

Draw a figure of measurement of the outline of your school land

- Find an approximate measurement of the land of your school.
- What is the area of unused land of the school?
- What is the portion of unused land of total area?

Present your data in the classroom measuring in the information.

Thoughout this learning experience, you completed various tasks about properties and construction of triangles and quadrilaterals. You can apply this experience to measure various objects and also to solve various mathematical problems.

# Exercise

1. To measure the area of the shape in the given figure- A, how will you use the properties of a right-angled triangle? Solve the problem and write the logic behind your solution.

   If AD = 12 cm, find the length of BC.

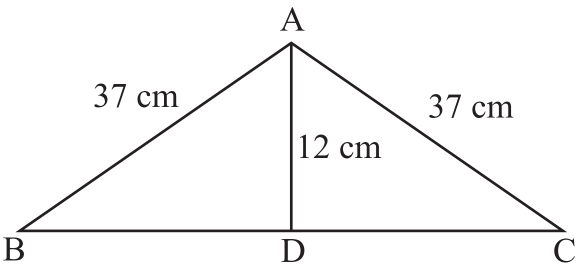



Figure- A

2. Prove that the diagonals of a square are equal. You may draw a figure or prove by cutting papers.

3. Suppose length of four sides are 4 cm, 3 cm, 3.5 cm, 5 cm and one of the angles is 60 degrees. Construct the quadrilateral.

4. In figure- B, AB = ?

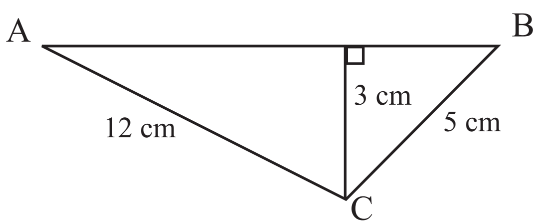



Figure- B

5. To color a wall of your school, supposethe base of a ladder of 15m is placed at 12m distance from the the wall then determine the height of the wall up to the tip of the ladder.

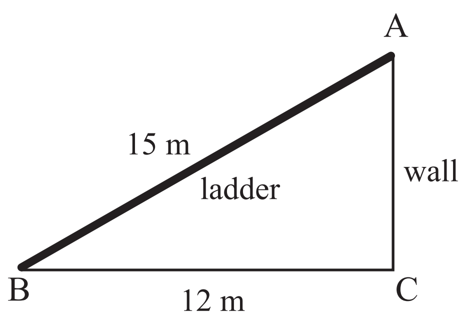



Figure- C

6. Calculate the perimeter of the given rectangle. (Figure- D)

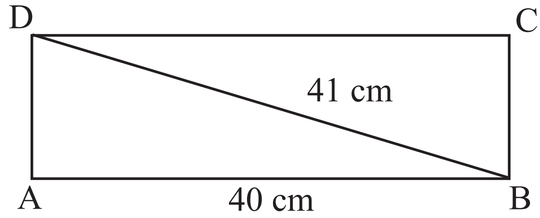



Figure- D

7. Suppose for a rhombus ABCD, the diagonals AC = 30 cm and BD = 16 cm. Calculate the perimeter of the rhombus.

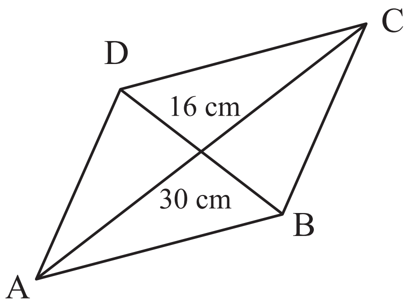



Figure- E

8. Check the validity of the statement, "If (3, 4 and 5) are Pythagorian triplets, then (3k, 4k and 5k) are also Pythagorian triplets, where k is any positive integer.

9. Verify the following statement by constructing a triangle or cutting papers, "The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and half in length."

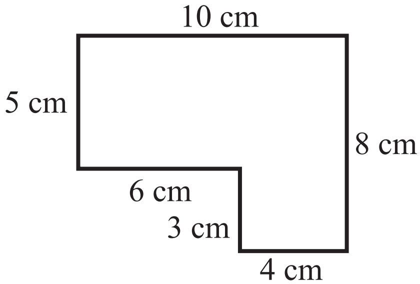
10. Suppose for a parallelogram the length of two adjacent sides are 6cm and 5cm and the angle between these sides is 50°. Construct the parallelogram.

11. Suppose the length of a side of a square is 5 cm. Construct the square.

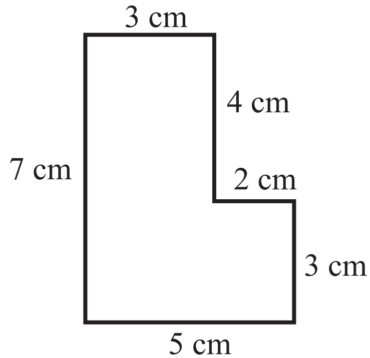
12. Suppose for a parallelogram shaped land the length of two adjacent sides are 4 m and 5 m and length of a diagonal is 7 m. Determine the area of the land.

13. For a rectangular land ABCD, AB = 10 m and diagonal AC = 16 m. If the intersecting point of the diagonals is G, calculate the area of ΔAGB.

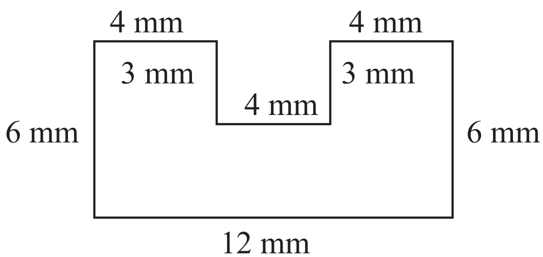
14. Measure the area of following shapes



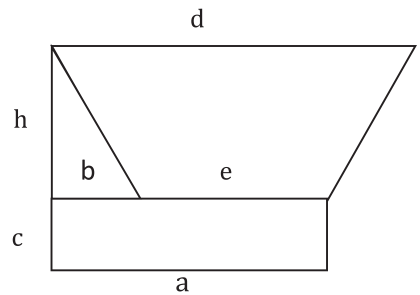

(a)




(b)




(c)




(d)

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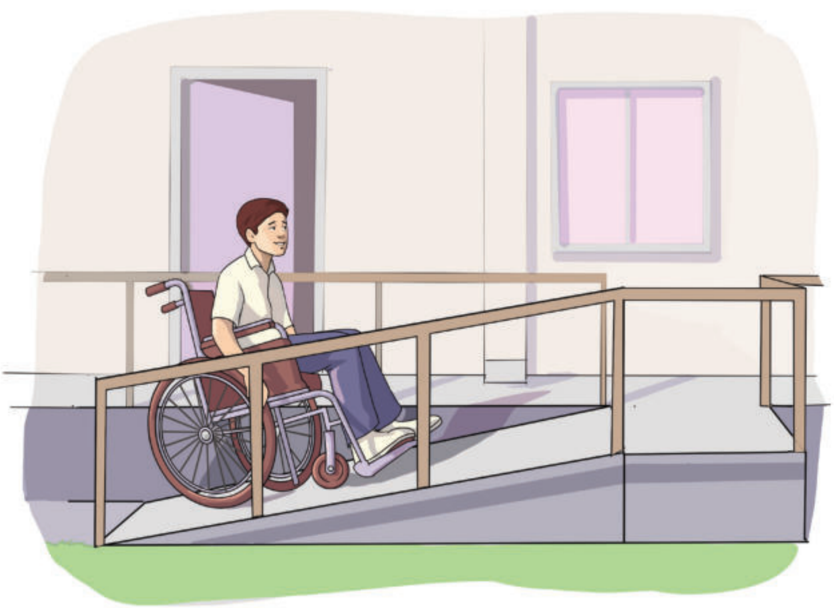
# Coordinate Geometry in location maps

## You can learn from this experience

- Cartesian coordinates
- Distance between two points using coordinates
- Midpoint of a line segment
- Equation of a straight line
- Slope of a straight line



# Coordinate Geometry in location maps

In our daily life, we use the map for various reasons: to find out geographical location of a place, locate historical places, measure lands, etc. Geometry has contributed a lot to making the map. We can locate any place very easily using coordinate geometry. In this experience, we will use coordinate geometry to locate different parts of our educational institution and make its map.

## An educational institute on the map

Below you can see the model map of an educational institute. There is the office building, hostel, garden, etc. In the box given, make a list of what else you can see.

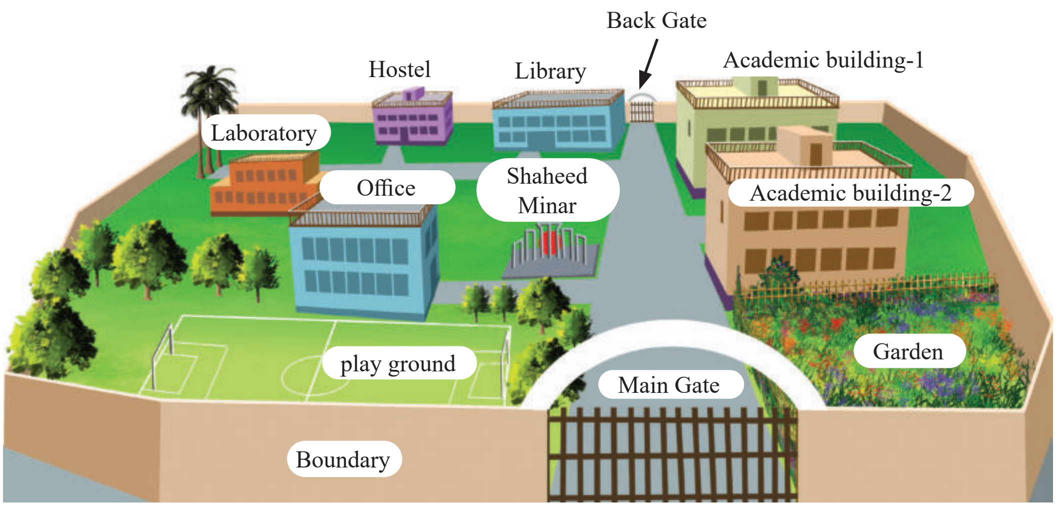



Figure- 6.1

Show this list to your teacher and classmates. By discussing with them, make corrections to the list.

The locations of the different areas of the institute, the garden, the play ground, and the trees in the map have been represented as points in figure: 6.2. A map like this is called **Location Map.**

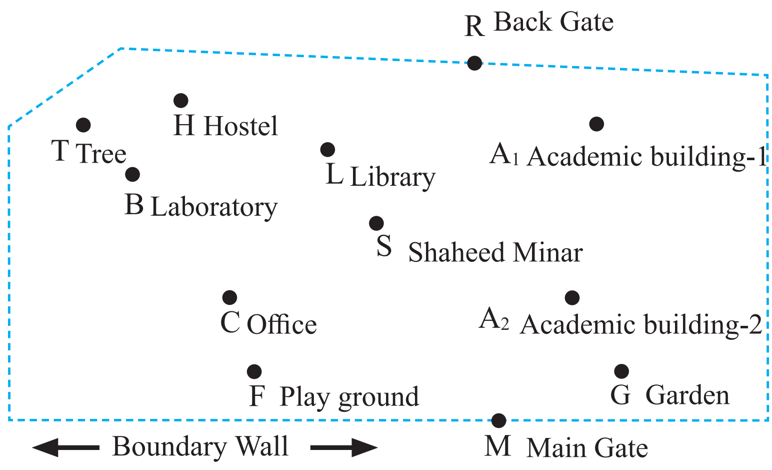



Figure: 6.2.

A location map can't be made by guessing. Mathematical calculations are required. The important things needed to make a location map are the accurate locations and sizes of various objects and the distance between the objects. Then the objects are drawn on the map with the proper measurements. We need coordinate geometry to do this task. Now, let's study the essential topics of this experience.

## Cartesian Coordinate System

To draw the map of an institute, we have to do some relevant calculations. For example, what's the land area of the institute? How long are the boundaries in the North, West, South, and East directions? How many buildings are there in the institute and what is in their neighborhood? How far is one building from the other? Is the pathway straight or curved? Where are the trees, garden, field, etc situated? To answer these questions, mathematical calculations are needed and then using a proper scale (proportion/ratio) a location map is drawn on paper. The coordinate system is used to achieve these objectives. To understand coordinate systems in detail, we will perform some tasks with the help of the picture below.

## Locating with coordinates

What do you see in Figure 6.3? The teacher is standing and the students are seated. Where the teacher stands, there is a horizontal number line and a vertical number line. These two number lines have names; the horizontal line is called the *x*–axis and the vertical line is called the *y*–axis. These axes are perpendicular to each other. The point where the two axes meet is called the origin. In Image 6.3, the axes intersect where the teacher stands. This intersection point is the origin. With respect to the origin, a teacher can accurately pinpoint a student's location. For example, with respect to the origin, student M is located 5 units away in the direction parallel to the *x*-axis. And parallel to the *y*-axis, he is 6 units away from the origin. In short, we write his location as M(5,6). Similarly, with respect to the origin, student G is 3 units away in the x direction and 4 units away in the y direction. In short, we write his location as G(3,4). Here the first number is called the Abscissa (*x* coordinate) and the second one is called the Ordinate (*y* coordinate). We can always express the location of something by this method: using the origin, *x* and *y* axes. Now, write down the coordinates of P.

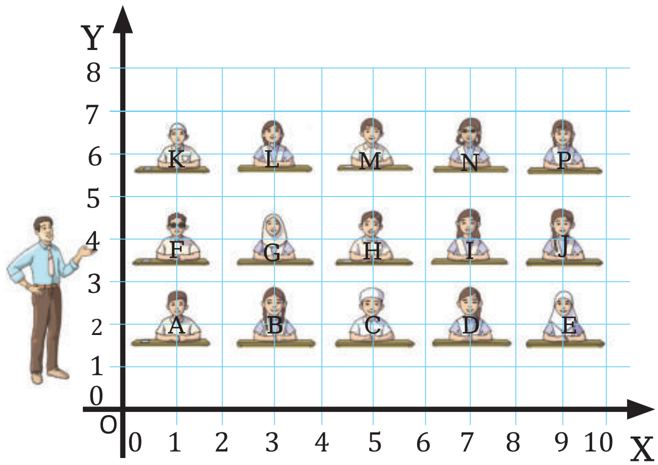



Figure: 6.3

## Individual Task

Using Figure 6.3, fill up the following table:

**Table 6.1**

| Name | Abscissa | Ordinate | Coordinate |
|---|---|---|---|
| B | 3 | 1 | B(3, 1) |
| G | | | |
| L | | | |

| Name | Abscissa | Ordinate | Coordinate |
|---|---|---|---|
| E | | | |
| K | 1 | 6 | |
| A | | | |

Notice that, in Figure 6.3, the locations of the students were found with reference to the teacher's location. Locating an object has to be done with respect to the location of another object. For example, the student's locations were found with reference to the teacher's location. Here the teacher's location is the origin. If you locate objects with respect to your location, then your location is the origin.

Both axes intersect at the origin. The abscissa and the ordinate are both 0 there. So, the coordinates of the origin are (0, 0). This mathematical system of locating objects using the origin, the $x$ & $y$ coordinates, is called Coordinate Geometry. French Philosopher, Mathematician and Scientist Rene Descartes invented it. Hence, this branch of Geometry is named Cartesian Geometry after him. It's a very important branch of Mathematics.

Rene′ Descartes

Suppose you want to go from the origin to the point P(5,4). Move 5 units along the $x$-axis and then move 4 units parallel to the $y$-axis. You reach point P(5,4). Likewise, if you move from the origin 4 units along the $y$-axis and then 5 units parallel to the $x$-axis, you again reach P(5,4). If you draw these two paths to P, you get a rectangle. That is, the axes and two perpendicular lines from P to the axes form a rectangle (See Image 6.5 ). This is why Cartesian Coordinates are also called Rectangular Cartesian Coordinates.

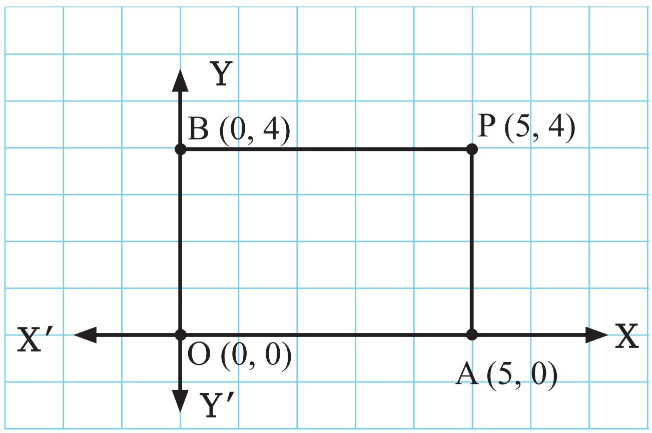



Figure: 6.5

In figure 6.3, the teacher was in a corner of the classroom. What does he do if he is at the center of the room and wants to locate the students? He can make a little change and easily find the student's location. He extends(stretches) the x-axis to the left. On this extended line, numbers to the left of 0 are negative. The numbers become smaller and more negative as we move left. Likewise, the teacher extends the y-axis downwards. On this extended line, the numbers below 0 are negative and they get smaller (more negative) as we move down.

## Individual Task

Find out the coordinates of the points in figure 6.6 from the origin and fill up the table:

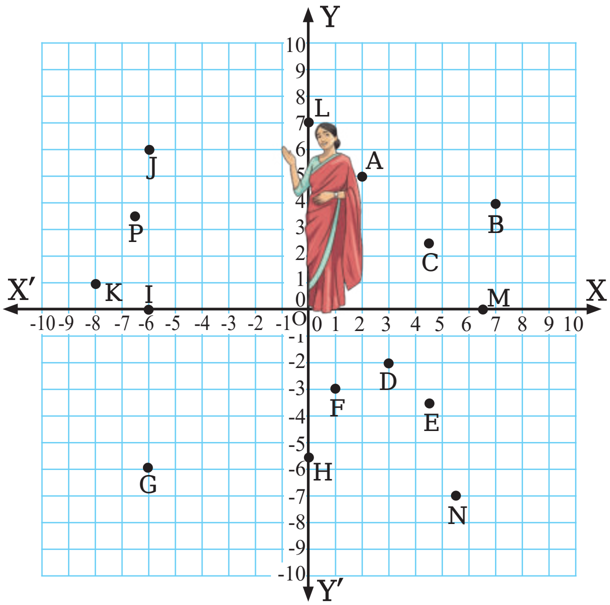



Figure 6.6

| Table 6.2 | | | |
|---|---|---|---|
| Name | Abscissa | Ordinate | Coordinate |
| K | −8 | 1 | K(−8,1) |
| C | | | |
| H | | | |
| N | | | |
| P | −6.5 | 3.5 | P(−<br>6.5,3.5) |
| M | | | |
| A | | | |
| B | | | |
| D | | | |
| E | | | |
| F | | | |
| G | | | |
| I | | | |
| J | | | |
| L | | | |

Note: The y coordinate of any point on the $x$-axis is 0, and the $x$ coordinate of any point on the $y$-axis is 0.

## Group Work:

In the graph (figure 6.7), choose an origin and draw the $x$ and y axes through it. Then plot the given points on the graph:

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A(−3.5, 5.5), B(−4, −4), C(0, − 5, 5),

D(−5,0), E(3.5, − 5.5 ) ,F(3.5, − 5.5 ), G(0, 1.5 )

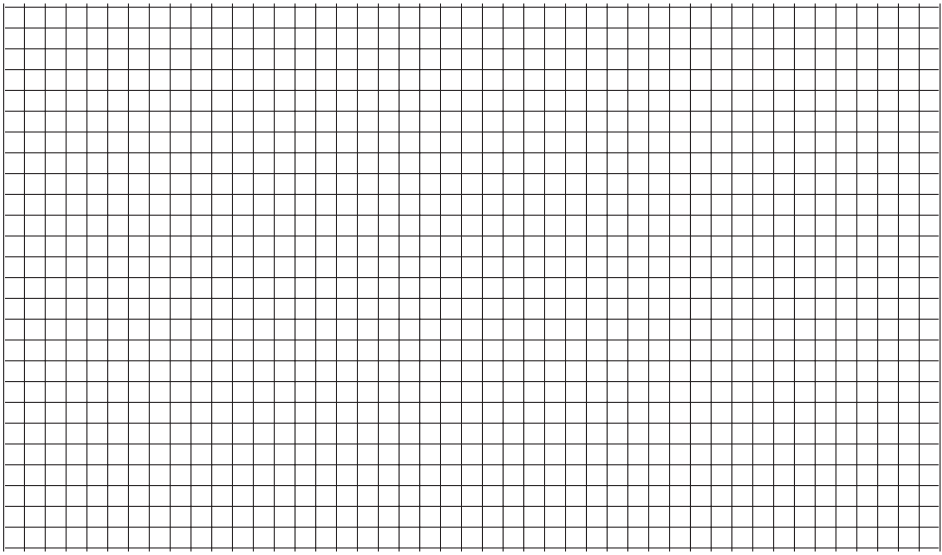


Figure: 6.7

## Quadrant

You have seen that the abscissa can be negative or positive and the ordinate can be negative or positive depending on the location. So we can divide the xy-plane into four regions. We call these regions the 1st quadrant, $2^{nd}$ quadrant, $3^{rd}$ quadrant, and $4^{th}$ quadrant. See figure 6.8. Can you tell, what will be the signs (+ ve or – ve) of the *x* and y coordinates in these quadrants? Fill up Table 6.3.

**Table 6.3**

| Quadrant | Sign of the *x* coordinate | Sign of the y coordinate |
|---|---|---|
| $1^{st}$ | | |
| $2^{nd}$ | | |
| $3^{rd}$ | | |
| $4^{th}$ | | |

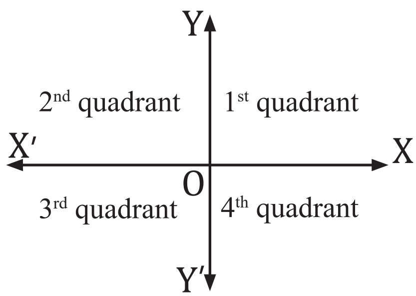



Figure 6.8

No matter in which quadrant a point is, its *x*-coordinate is the perpendicular distance from the *y*-axis to that point and its *y*-coordinate is the perpendicular distance from the *x*-axis to that point. Then, depending on the quadrant, *x* and *y* coordinates are given the appropriate sign (+ or – ).

## Distance between two points

To draw a map, we measure the distance between various objects. Then, using a proper scale (for example an actual length of 1km is chosen to be 1cm on the graph), we plot the positions of the objects on the graph. Now we will calculate the distance between two points on a graph using Pythagoras' Theorem.

## Finding distance using Pythagoras' theorem

P(3, 4) and Q(9, 7) are two points on the *xy*-plane, see figure 6.9. Point R lies directly below Q(9, 7). The line segment PR is horizontal and the line segment QR is vertical. The triangle PQR is right-angled at R.

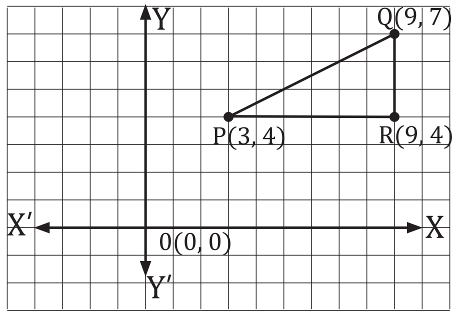



Figure 6.9

Now think about point R. What are its coordinates? The line segment QR is parallel to the *y*-axis. Hence the *x*-coordinate of R is 9 (because moving along a vertical line does not change the *x*-coordinate). Similarly, the line segment PR is parallel to the *x*-axis. Hence, the *y*-coordinate of R is 4 (because moving along a horizontal line does not change the *y*-coordinate). So (9, 4) are the coordinates of R. The y coordinates of P and R are the same, so the length of the line segment PR is just the difference of the *x*-coordinates of P and R, that is PR = 9 – 3 = 6. Likewise, RQ = 7 – 4 = 3.

Now, applying Pythagoras' theorem: $PQ^2 = PR^2 + RQ^2$

Therefore, $PQ = \sqrt{PR^2 + RQ^2}$ [Note: Distance can not be negative]

$$= \sqrt{(9-3)^2 + (7-4)^2}$$

$$= \sqrt{36+9}$$

$$= \sqrt{45}$$

$$= 3\sqrt{5}$$

So, the distance between P and Q is $3\sqrt{5}$ units.

## Individual Task

There are some points in the following table. Determine the difference between the points.

**Table 6.4**

| First Point | Second Point | Difference in x coordinates | Difference in y coordinates | Distance |
|---|---|---|---|---|
| B(−8, 4) | O(0, 0) Origin | −8 −0 = −8 | 4 − 0 = 4 | $\sqrt{(8)^2+(-4)^2} = \sqrt{64+16} = 4\sqrt{5}$ |
| A(4, 6) | B(−8, 4) | | | |
| B(−8, 4) | C(2, 3) | | | |
| D(2, - 3) | E(−3 ,2) | | | |
| F(-5, - 6) | A(4, 6) | | | |

If, P($x_1$, $y_1$ ) and Q($x_2$, $y_2$) are two points,then the distance PQ is

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(\text{Difference in } x \text{ coordinates})^2 + (\text{Difference in } y \text{ coordinates})^2}$$

## Midpoint of a line segment

Sometimes we need to find the middle(center/midpoint) between two points on a map. For example, we might need to find the midpoint of a large building or a field and plot it on a graph. Let's see how coordinate geometry helps us to do that.

**If the two points lie on the axes**

Suppose, $P(x_1,0)$ and $Q(x_2,0)$ are two points on the *x-axis*

So, distance $OP = x_1$ and $OQ = x_2$

Let $R(x,0)$ be the midpoint of the line segment $PQ$.

So $OR = x, PR = OR - OP = x - x_1$

and $QR = OQ - OR = x_2 - x$ .

Since R is the midpoint of PQ, PR = QR.

So, $x - x_1 = x_2 - x$

or, $2x = x_1 + x_2$

or, $x = \dfrac{x_1 + x_2}{2}$

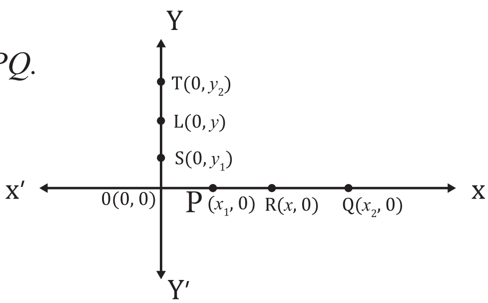



Figure 6.10

R is a point on the $x$-axis, so its $y$-coordinate is 0. Hence R has coordinates $R\left(\frac{x_1+x_2}{2}, 0\right)$.

Now consider two points on the y-axis S$(0, y_1)$ and T$(0, y_2)$.

Here OS = $y_1$ and OT = $y_2$.

Suppose the midpoint of the line segment ST is L.

Let L has coordinates $(0, y)$. Can you prove that L has coordinates $L\left(0, \frac{y_1+y_2}{2}\right)$ ?

## If the two points lie anywhere

We have learned how to find the midpoint of a line segment when the points lie on the axes. Now we will look at the case where the points can lie anywhere on the xy-plane.

Let P$(x_1, y_1)$ and Q$(x_2, y_2)$ be two points on the $xy$-plane. We draw a line segment PL so that PL is parallel to the $x$-axis and draw another line segment QL such that QL is parallel to the y-axis. You can easily see that PL and QL intersect at point L. Therefore, PQL is a right-angled triangle.

Let A be the midpoint of PL. We draw the line segment AR, perpendicular to PL. Notice that LQ and AR are parallel.

Also, let B be the midpoint of LQ, and we draw the line segment BR, perpendicular to LQ. Notice that PL and BR are parallel. Now write down the coordinates of R below:

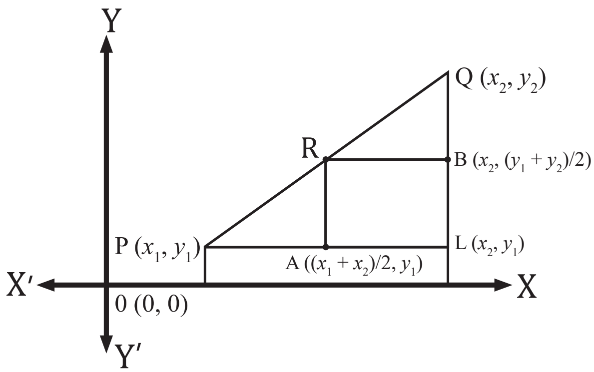



Figure 6.11

If we consider the right-angled ΔPAR and ΔBRQ, it's easy to see that AP = AL = BR, and AR = LB = BQ. Hence, the triangles are congruent. So PR = RQ, which means R is the midpoint of PQ. Therefore, R has coordinates $R(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

Midpoint Formula: If P$(x_1, y_1)$ and Q$(x_2, y_2)$ are two points and R is the midpoint of the line segment PQ,
then R has coordinates $R(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

So, Midpoint of two points = $\left(\frac{\text{Sum of two abscissa}}{2}, \frac{\text{Sum of two ordinates}}{2}\right)$

**Example:** Find the midpoint of the line segment AB where A(4, 6) and B(−8, 4).

**Answer:** Midpoint of AB = $\left(\frac{4+(-8)}{2}, \frac{6+4}{2}\right) = \left(\frac{4-8}{2}, \frac{10}{2}\right) = (-2, 5)$

## Individual Task

Fill up the table below:

**Table 6.5**

| Number | First point | Second point | Midpoint |
|---|---|---|---|
| 1 | $B(-8,4)$ | $O(0,0)$ | $\left(\frac{-8+0}{2}, \frac{4+0}{2}\right) = (-4, 2)$ |
| 2 | $A(4,6)$ | $B(-8,4)$ | |
| 3 | $C(-5,-5)$ | $D(6.5,-6.5)$ | |
| 4 | $B(-8,4)$ | $D(6.5,-6.5)$ | |
| 5 | $A(4,6)$ | $C(-5,-5)$ | |
| 6 | $B(-8,4)$ | $C(-5,-5)$ | |

## Group Work

A school map is shown below. Observe it carefully.

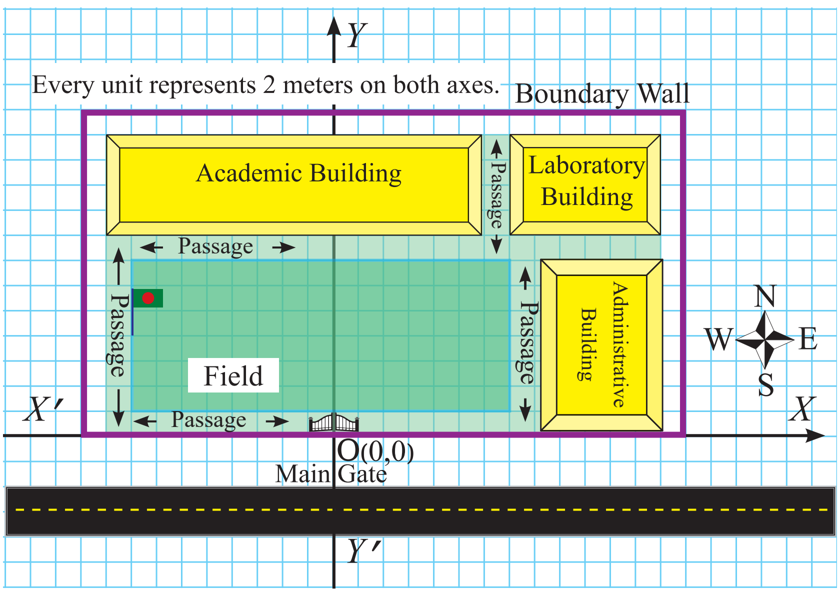



Figure 6.12

Now, as per your teacher's instructions fill up the table 6.12 below. Take the middle of the main gate to be the origin.

| Table 6.12 | | |
|---|---|---|
| Number | Question | Answer |
| 1 | Find out the coordinates of the bottom of the flagpole. | |
| 2 | Write down the coordinates of the four corners of the field. | |
| 3 | Find out the length of the field. | |
| 4 | Find out the midpoint of the laboratory building. Then find the distance of this point from the main gate. | |
| 5 | Find out the coordinates of the midpoint of the administration building. Also, work out the distance between this building and the laboratory building. | |
| 6 | For the field, find the length of the diagonal. | |
| 7 | What's the midpoint of the field? | |
| 8 | Find the perimeter of the school's boundary wall. | |
| 9 | Make two challenging questions for a friend/classmate from the map. | |

## slope

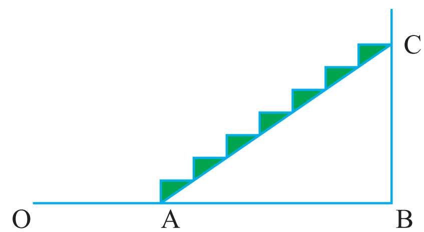
Have you ever seen the slope of the riverbank or that of a mountain? You have definitely seen the staircase of your school. As seen from the plain land, the riverbank gradually inclines(falls). But the hills and the staircase gradually incline(rise). This inclination of a line with respect to the plain land is called the slope. In simple words, a slope is nothing but the gradual incline of something.


Figure 6.13 slope of river bank


Figure 6.14 slope of mountain


Figure 6.15 Slide

In coordinate geometry, how much a straight line inclines with respect to the +ve $x$-axis is the Slope/Gradient. In other words, the slope of a straight line is nothing but the measure of how steep it is. It's mathematically defined to be the ratio of rise (vertical distance) over run (horizontal distance). Put another way, if you move 1 unit horizontally on a straight line, the corresponding rise/fall in the vertical distance is the Slope. For the staircase below, the horizontal distance (run) is AB and the vertical distance (rise) is BC. Then,



Fugure 6.16

$$\text{Slope} = \frac{BC}{AB} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

## Finding gradient using coordinates

If we take two points P($x_1$, $y_1$) and Q($x_2$, $y_2$ ), then we can find the gradient (slope) of PQ from the picture below:

Here, the gradient of

PQ

$$= \frac{RQ}{PR} = \frac{y_2-y_1}{x_2-x_1} = \frac{\text{Difference in } y \text{ coordinates}}{\text{Difference in } x \text{ coordinates}}$$

We usually denote gradient by the letter m.

Hence gradient, $m = \frac{y_2-y_1}{x_2-x_1}$

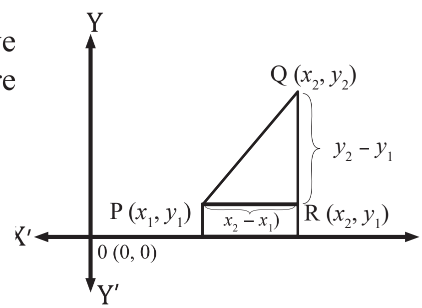


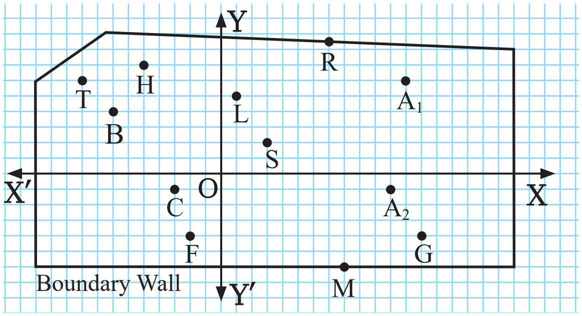
Figure 6.17

The gradient/slope of a line can be positive or negative. If the line rises (inclines) in the

+ve $x$-direction, then the slope is +ve. If the line falls (declines) in the +ve x-direction, then the slope is –ve.

## Group work

The concept of slope is used in location maps too. The different portion of the school premises is represented by points in the coordinate plane (see the image above). Answer the following questions:




চিত্র: ৬.১৮

1) Find the slope of the straight line passing through the following points:

   a) S and $A_1$ b) S and $A_2$ c) C and G d) F and T

2) Choose any three straight lines through any three pair of points and find their slopes.

## The equation of a straight line

### Equations of horizontal lines (Lines parallel to $x$-axis)

If we represent the students in Image 6.3 by points, then we get the image below.

If we represent the students in Image 6.3 by points, then we get the image below.

The points are denoted by letters. If you observe the points A(1, 1), B(3, 1), C(5, 1), D(7, 1) and E(9, 1), do you notice domething?

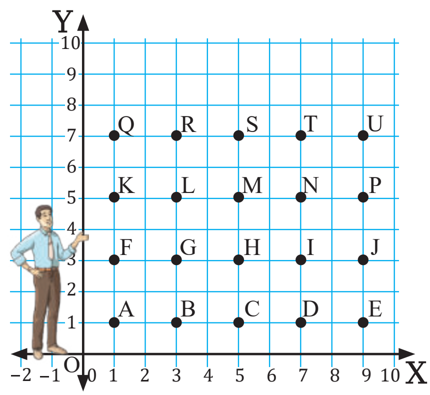



Figure 6.19

Write down your observations here:

You might have noticed that all the points have the same $x$ coordinate. And if you join these points serially you get a straight line. The interesting fact is, the ordinates of all the points on this line are the same and is equal to 1. Algebraically, we write the equation of this line as: $y = 1$.

This line is parallel to the $x$-axis.

## Individual Work

According to Image 6.20, solve the following:

1. Find the equation of the straight line connecting the points F(1, 3), G(3, 3), H(5, 3), I(7, 3), J(9, 3)
2. Find the equation of the straight line connecting the points A, B, C, D and E.
3. Find the equation of the straight line connecting the points K, L, M, N and P.
4. Find the equation of the straight line connecting the points Q, R, S, T and U.

Can you tell what the ordinate of a point on the $x$-axis is? Any point on the $x$-axis has $y$ coordinate equal to 0. So, the equation of the $x$-axis 0.

Now think about the points (3, −4), (5, −4), (7, −4). What's the equation of the straight line connecting them? Notice that the ordinates are the same for these points, although negative. All the y coordinates are – 4. Write down the equation of the line below:

## Pair Work

Joining which sets of points bellow do we get straight lines parallel to the x-axis? Write them following box.

### Given are the points

A(3, −3), B(4, 4), C(−6, 4), D(−4, 7),

E(−8, 4), G(-10, −3), H(12,17),

I(13, −3), J(15, 7), K(17, 3),

L(18, 4), M(20, 7)

Plot the points on the graph given (figure 6.20). Connect these points so that you get straight lines parallel to the x-axis. Match your answers above with this graph:

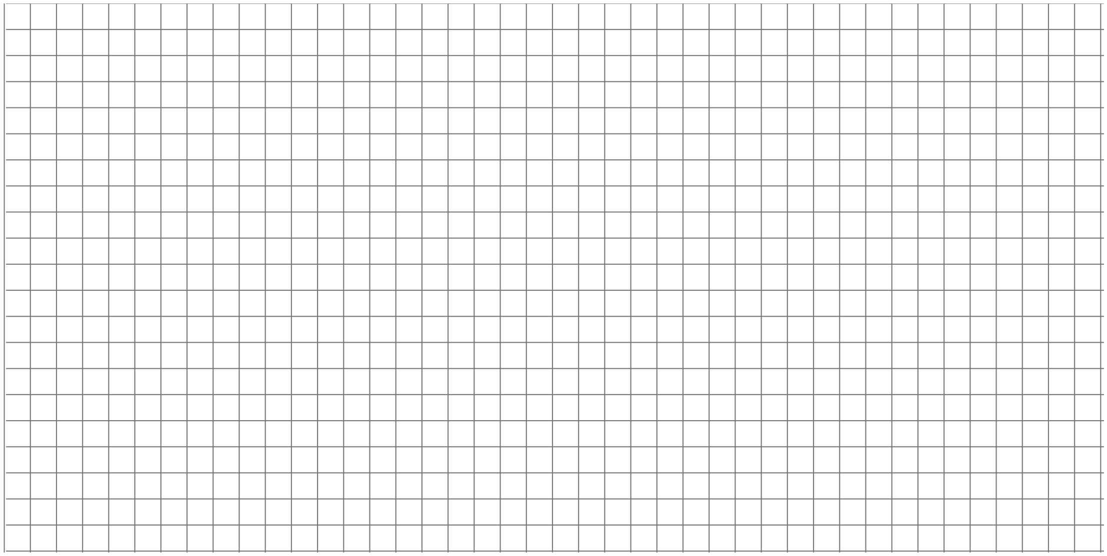


Figure 6.20

From the above observations, what can you conclude? We can conclude that:

**If we join the points whose ordinates are the same, then we get a line that is parallel to the *x*-axis.**

## Equations of vertical lines (Lines parallel to *y*-axis)

Just like the equations of lines parallel to $x$-axis, we can find the equations of the lines parallel to $y$-axis. From Image 6.20, write down the coordinates of the points A, F, K and Q:

Notice that the $x$-coordinates of these points are all 1. If you join these points, you get a straight line. What can you say about its equation? All of the points on this line have the same $x$-coordinate, so its equation is $x = 1$. It's easy to see that this line is parallel to the $y$-axis.

## Group Work

Joining which sets of points bellow do we get straight lines parallel to the y-axis? Write them down:

Given are the points

A(−3, −3), B(4, 4), C(3, −6), D(7, 7),

E(4, −6), G(7, −3), H(4, −7), I(-3, 8),

J(7, 12), K(4, 11), L(7, 4), M(−3, 0),

N(0, 6), P(0, −6)

Plot the points on the graph given (Figure 6.21). Connect the points so that you get straight lines parallel to the *y*-axis. Match your previous answers with this graph.

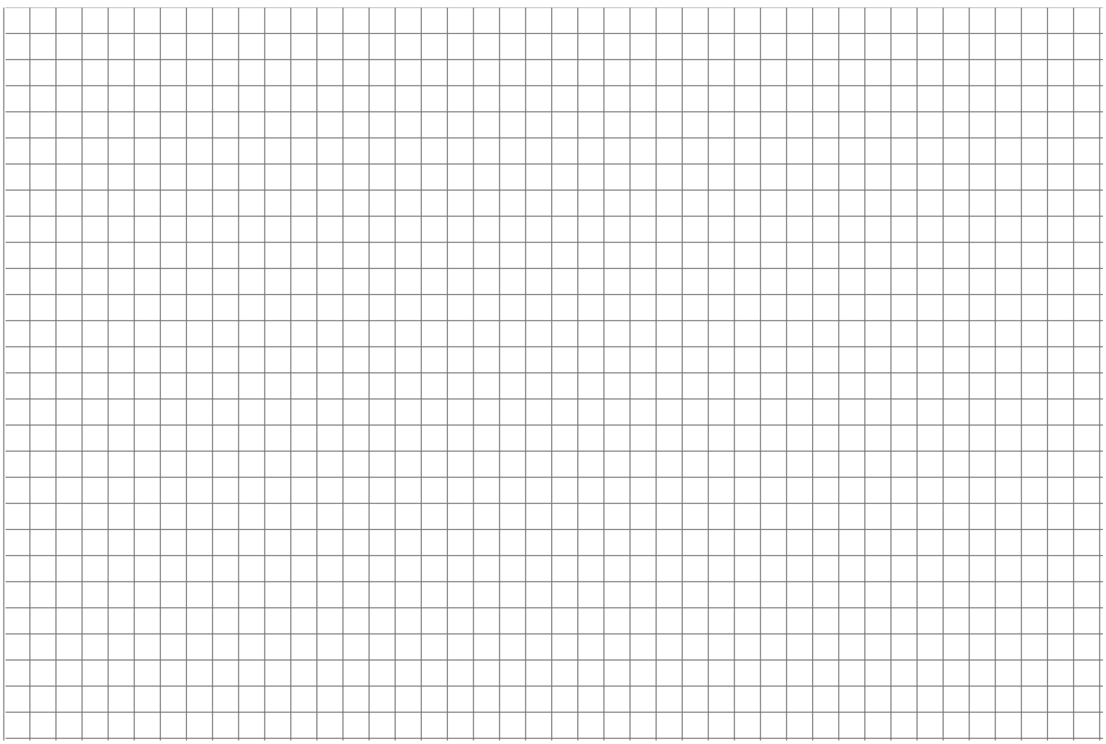


Figure 6.21

What is the equation of $y$-axis? Write your answer below.

From the observations above, what can you conclude? We can conclude that:

**If we join the points whose abscissas are the same, then we get a line that is parallel to the $y$-axis.**

## Pair Work

Draw the following lines: $x = 6$, $x = -5$, $y = 3$ and $y = -4$. The lines form a rectangle. Find the coordinates of the corners of this rectangle and find its area.

### Equation of straight lines not parallel to the axes

Let's find the equations of straight lines that are not parallel to the axes. In the location map (figure 6.22), we have to find the equation of the straight line that passes through S(3, 2) and $A_1$ (12, 6).

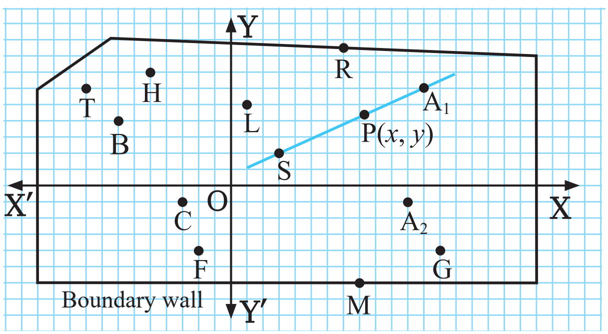



Figure 6.22

Suppose P(x, y) is any point on the line $SA_1$. Now, the gradient of the line SP $= \dfrac{y-2}{x-3}$.

Also, the gradient of the line $SA_1 = \dfrac{6-2}{12-3} = \dfrac{4}{9}$

Since $SA_1$ and SP are basically the same straight line, their gradients are equal.

So , $\dfrac{y-2}{x-3} = \dfrac{4}{9}$

Or, $4x - 12 = 9y - 18$

Or, $4x - 9y - 12 + 18 = 0$

Therefore, $4x - 9y + 6 = 0$ is the equation of $SA_1$.

## Pair Work

From Image 6.22:

1) Find out the equation of the straight line that passes through the origin and the point $A_2$
2) Find the equation of the straight line CL.

## General Equation of a Straight Line

Let us now find the equation of any straight line. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points on the $xy$-plane. We will find the equation of the straight line AB. If $P(x, y)$ is a point on the line AB, then:

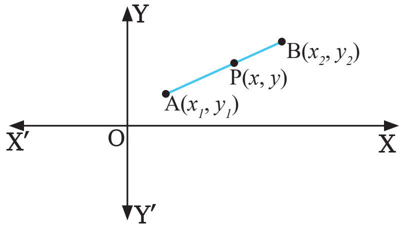


Figure 6.23

Gradient of AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

Also, Gradient of AP $= \dfrac{y - y_1}{x - x_1}$

Since AB and AP are basically the same straight line, gradient of AP = gradient of AB. That is,

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Or, } \frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\therefore \frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

This is the equation of the straight line that passes through A and B.

The equation of the straight line passing through the points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

## Individual Work

1. Find the equation of the straight line that passes through the points (3, 4) and (2, –3).
2. Find the equation of the straight line that passes through the points (0, 0) and (–7 , –3)

## Finding the equation of a straight line using the slope

We know the equation of a straight line is

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

Or, $$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

Or, $$\frac{y - y_1}{x - x_1} = m \text{ [since } m = \frac{y_1 - y_2}{x_1 - x_2} \text{ is the slope]}$$

$$\therefore y - y_1 = m(x - x_1)$$

> The equation of a straight line passing through the point $(x_1, y_1)$ with gradient $m$ is:
>
> $$y - y_1 = m(x - x_1)$$

## Individual Work

Find the equation of a straight line with gradient 3 that passes through the point (0, 1).

## Group Work

The location map of the school in the beginning of the experience is shown in the graph below. With your teacher's instructions, form a group and solve the following problems:

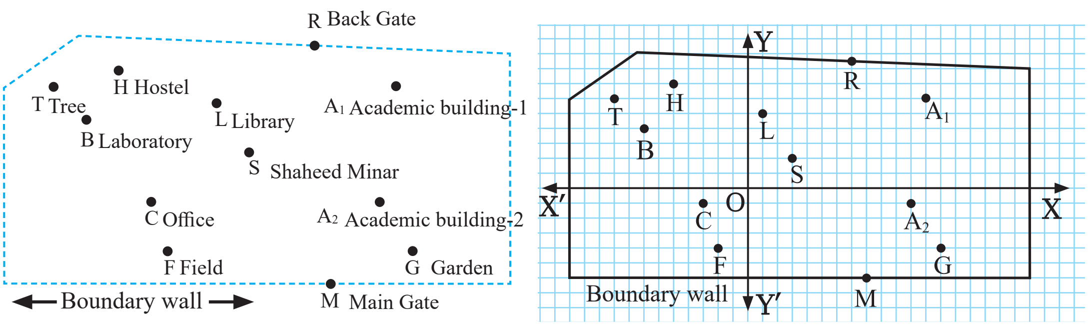



Figure 6.24

a. Find the coordinates of the laboratory, library, hostel and main gate.

b. In which quadrants are Academic Building 1 and the field situated?

c. What's the equation of the straight line that passes through the hostel and the shaheed minar?

d. How far is the hostel from the main gate?

e. Which part of the school is the farthest from the main gate?

f. Which gate is the closest to the hostel? Give your reason.

g. Compare the slope of the straight line that passes through the office building & shaheed minar with the slope of the line that passes through the library & back gate.

h. What's the total length of the boundary of the school?

i. Find the equation of the straight line that passes through the field and office building.

## Project Work

**As per your teacher's instructions:**

1. Make a location map of your school on a graph.
2. Choose a convenient part of the school as the origin and indicate the axes.
3. Choose any convenient part of the school and locate them on your map.
4. Find the distance between the two closest places in the school and the two farthest places in the school.
5. Find the coordinates of the headmaster's office room.

With your teacher's instructions, choose a specific date and exhibit your project in the school.

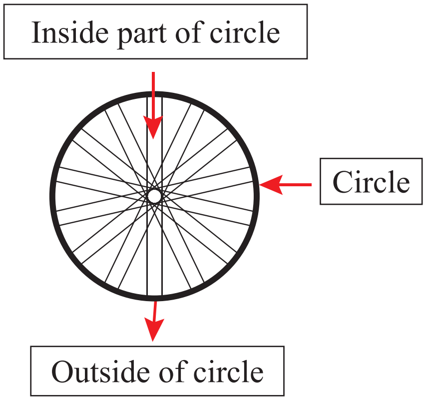
## Exercise

1. Plot the points A(2, 2), B(10, 1), C(11, 9) and D(3, 10) on a graph paper. Draw the lines AB, BC, CD and AD. What type of quadrilateral do you get? Give reasons.
2. Three points have coordinates A(−2 , 1), B(10, 6) and C(a, −6). If AB = BC, find the possible values of 'a'. For each possible value of 'a', find the corresponding area of the triangle ABC.
3. Find the equation of a straight line with gradient −2 that passes through the point (4, −5)
4. Find the equation of a straight line that passes through the points A(3, −3) and B(4, −2). What is the gradient of the straight line?
5. Plot the points A(2, 2), B(10, 1), C(11, 9) and D(3, 10) on the graph paper and draw the lines AB, BC, CD, AD. What kind of area is formed by these lines? Give reason in favour of your answer.
6. The coordinates of three points are A(−2, 1), B(10, 6) and C(a, −6). If AB = BC, then determine the possible values of a. Find the area of triangle ABC formed for each value of a.
7. The coordinates of four points are A(−1, 1), B(2, −1), C(0, 3) and D(3, 3). Determine the area of the quadrilateral formed by the points.

# Ins and outs of circles

## You can learn from this experience

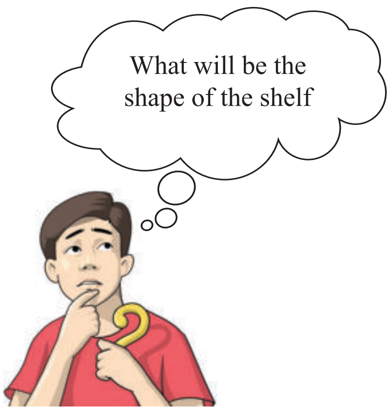
- Arcs
- Varius parts of circular area and circle
- Inscribed angle and central angle
- Various properties related to circle
- Quadrilateral circumscribed in a circle
- Tangent of a circle
- Measurements related to circle



# Ins and outs of circles

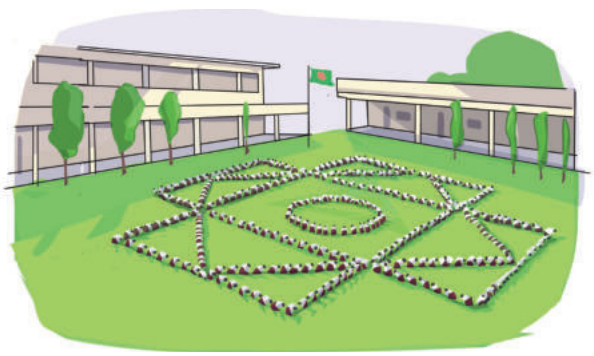
Suppose you want to arrange your study room nicely. So you planned to keep furnitures of different shapes in different spots of your room. You kept your circular reading table at a corner such that at one side it is with the window and at another side it is with the wall.

But the surface of your reading table is circular, so the corner has still some space between your table and two walls. Due to this some things often falls through that empty space. So if there were an open shelf at that empty spot, you could keep some small things like gifts in that shelf close to you, isn't it? You want to keep more than one steps in that shelf. So the steps must be of a shape whose front face is circular but the rear end fits with the corner of the room. The problem is, you don't have any idea about these shapes.।




So, we need to learn about these shapes. For that, we need to learn about ins and outs of circles, their properties, the mathematical relation among the porperties and the measurements etc.

You have seen geometric objects of various shapes at your school, your daily tools, the road you cross everyday. Some of the objects are natural and some are man-made. For example, for annual sports and various national holidays you make various displays.

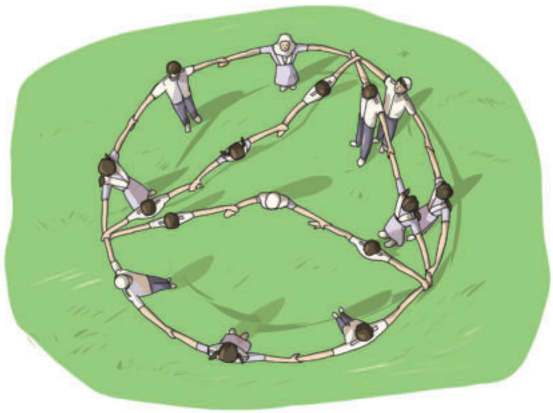


These are done with combination of geometric shapes, right? Let's make a display of our national flag (with stand) in our school field. But how do we do this?

Here, we have to know about rectangular and circular shapes by the ratio of the flag. We have already studied about the rectangular shapes. Now we will try to learn about circles by showing a display.

Circle is a planar geometric shape in which the points maintain a fixed distance from a fixed point. From this fixed point, if another point maintains a fixed distance, then the path created by the moving point is called a circle. The fixed point is called the center of the circle and the distance of any point on the circle from the center is called the radius of the circle.

**Direction for team display-** First 15-20 students make a circle by holding hands. Another student will stand at the center, Rest of the students create and display the radius, the diameter, chords ,etc. parts of the circle by holding hands and spreading fully. Discuss among yourselves and try to know which parts of the circle are created and what can be the relations among them. If needed talk to your teacher. Now display our beloved flag with stand.



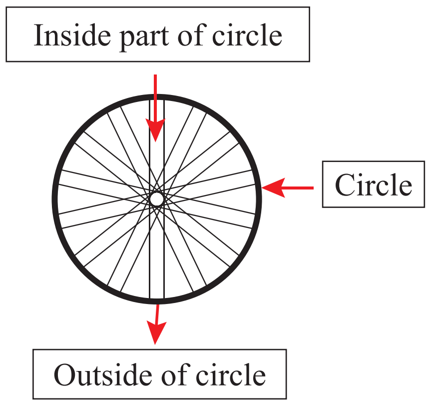
## Why do we need to learn about diameter and chords of a circle?

Do you understand anything from the given figure? Why is the handle of the bucket attached along the diameter of the open end? If we did not do this, would there be any problem? How do you carry a bucket filled with water if there is no handle? You

can also see many examples around us where we have handle along the middle of the open end.

> Write down the names of some tools you use or see where there are handles along middle of the open end to carry you have to hold in the middle while using that.

For better understanding of the concept shown above, you can play another game. For this you need circular rings of various size (key ring, bangles, plate, etc), old calendar or art paper or cork sheet, scale, tape, scissors, pin, rubber, and some sticks of different lengths. The game is – to observe which parts of a circle we obtain by placing sticks inside a circular ring in various ways and creating a model. Let's get to work.



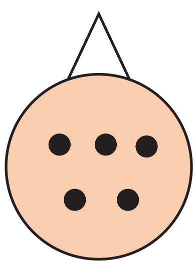

Rafiq placed a basketball ring on the backside of an old calendar and created a circle. Then he completed the figure by drawing a cycle wheel. The wheel divides the calendar page into three parts.

a) Circle (The wheel along with the steel frame)

b) Inside of the circle (The part where we have the spokes of the wheel)

c) Outside of the circle (The part on calendar outside of the wheel)

The region formed by the circle and it's inside called **Circular region.**



Ahona makes a circular region placing a ring on a thick cork sheet. Then she separates the circular disc from the cork sheet. Then she attaches some hooks and nails like the given figure. She plans to keep her keys hanging on this by placing the ring holder on the wall.

Amiya makes a circle on an art paper by putting a plate on it. But she does not know the center. Do you know how to identify the center? Amiya does it by cutting the circle from paper and making two equal folds. Now she puts the ring on top of a circular paper and fixes some straight stick of various lengths on that paper using tape. Then she marks the endpoints of the stick using the letters A, B, C, D and E. One end of the sticks are at point A and other ends are respectively at B,C,D and E to create the lines AB, AC, AD and AE. Out of them AC passes through the center. Also there is a smaller stick PQ which is entirely inside the circular region. It does not touch the circle at any point. Amiya measures the lengths of the sticks and write them in her notebook. Think and tell, which stick would be the longest? Now draw the model on your notebook using ruler, compass, pencil. Then put a tick mark in the corresponding box of the following table. Give your logic behind your answer.

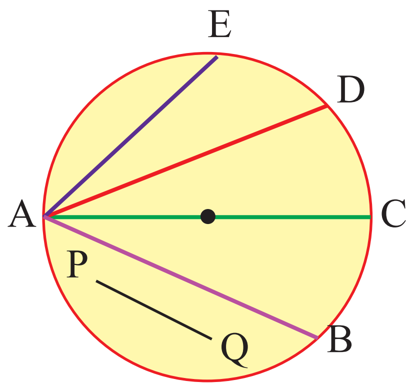



**Table 7.1**

| Segment | Chord | Diameter | Logic |
|---|---|---|---|
| AB | √ | | Did not pass through the center. |
| AC | | | |
| AD | | | |
| AE | | | |
| PQ | | | |

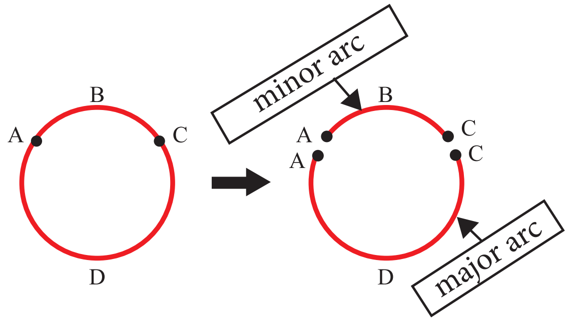
**Looking for relations among the line segments.**

a) The diameter of a circle is also a ............... But any chord of a circle is not.............

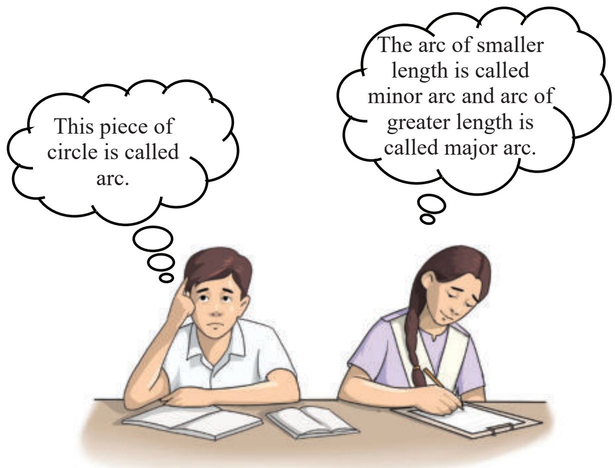
b) The diameter of a circle is the ............... chord.

## Arc

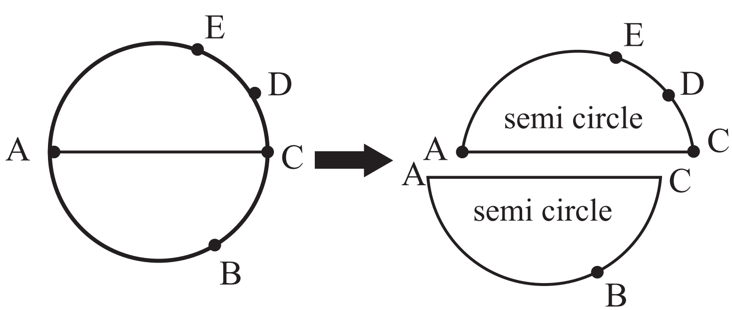
Mita draws a circle by keeping a bangle on paper. After she draws the cicle, the bangle falls and breaks into two pieces. She was not become sad, and picked up the pieces to draws a figure like the given one. She labels the smaller piece as ABC and the larger piece as ADC. What can we call these parts?




**If the parts were of same size, what would we call each of them?**



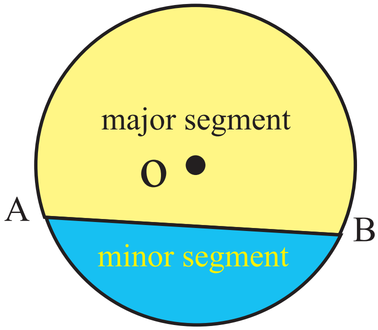

To answer this, let's observe Amiya's model again. In that model, AC was the diameter of the circle where the points A and C were on the circle. Now taking a piece of thread we can measure the lengths of the arcs AEDC and ABC separately. AC divided the circle into two equal pieces, isn't it? In that case each piece is called



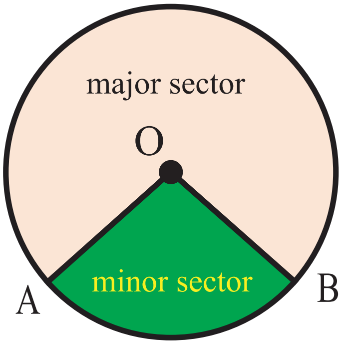

a **semi circle.** We also know that the lotal length of a circle is called the Circumference.

Let's learn about other parts of the circle. Draw two circles on paper by keeping the roll of tape you brought. Now cut the paper and separate the circular regions. Fold twice to find the center O. In one circle, draw a chord AB which is not a diameter. Notice that it divided the circle into two unequal parts. What is the name of each part? We call them **segment.**

Do they look equal? No, one is larger and the other is smaller. The larger part is called **major segment** and smaller one is called a **minor segment.** Use two colors of you choice to color the segments.

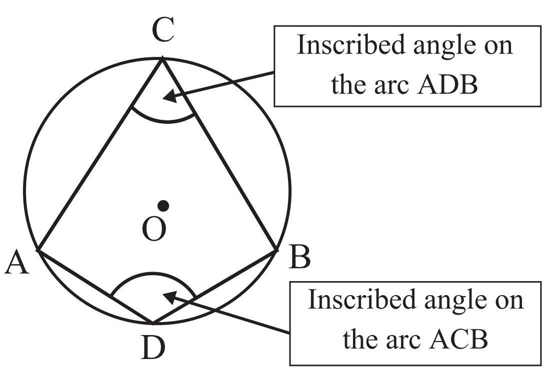



In the other circular region, draw two radii OA and OB like the figure. Now can you say what might be the name of the area bounded by the radii and the arc AB? We can call them Sector. The larger one is called major sector and smaller one is called minor sector.




**If the chord AB is a diameter, how will the segements look and what will be the name? Decide by drawing a figure.**

## Inscribed Angle




Draw a circle with a pencil-compass on a cork sheet. Mark the center of the circle as O. Now place two sticks in such a way that one end of them is together on the circle at point C and the other end is at point A and B on the two circles. Note that ∠ACB is created on opposite sides of the arc ADB. This ∠ACB is an inscribed angle on the arc ADB. Place two more sticks as shown in the figure in such a way that one end of them is together at point D on the circle and the other end is at point A and B on both circles. In this case ∠ADB is created on opposite sides of the arc ACB. This ∠ADB is another inscribed angle which is on the arc ACB. What can we call the other two angles ∠CAD and ∠CBD?

## Individual Task:

How many inscribed angles can be formed on an arc of a circle with sticks of different lengths? If more than one inscribed angle can be made, measure the angles using protractor. If necessary, make several circles of different radii on your notebook. For each circle, draw multiple inscribed angles on the same arc. Measure and observe the angles. Then fill in the blanks:

a) Angles inscribed on a minor arc of a circle are .................

b) Angles inscribed on a major arc of the circle are ................

c) The sum of the inscribed angles on minor and major arcs of the circle having same endpoints is ............... right angles.

d) Angles ins on the same arc of a circle are ………………

Write the difference of following topics in the box given at right side:

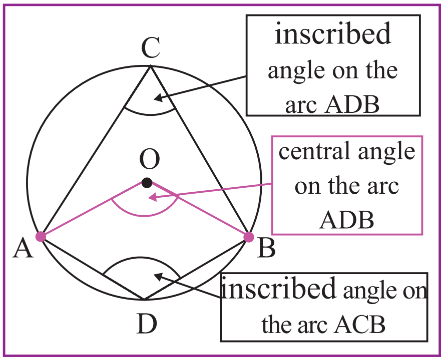
a) Minor arc and minor segment.

b) Major arc and major segment.

c) Segment and sector.

## Central Angle

Let's see what else can be done using the model that has been made to show inscribed angles. Now take two more sticks. Place the two sticks in the circle in such a way that one end makes an angle at the center point O of the circle and the other ends are at points A and B on the circle. In this case, what will be the length of the two sticks? Notice that, the ∠AOB is created on the arc ADB with vertex at the center. This ∠AOB is the central angle subtended on the arc ADB which is a minor arc of the circle.



**Measure with the help of a protractor and fill in the blanks below:**

a) On the minor arc ADB, the central angle ∠AOB =............... degrees and ∠AOB is an ................ [ acute/obtuse] angle. But the inscribed angle on the same arc is ..............., its magnitude is ...............degrees and the angle is an ................ [acute/obtuse] angle.

b) Now observe the central and inscribed angles at minor arc ADB and what do you decide? Central angle is ............. of inscribed angle or inscribed angle is ............. of central angle.

c) On the major arc ACB, the central angle ∠AOB =............... degrees and ∠AOB is an ................ [ acute/obtuse] angle. But the inscribed angle on the same arc is ..............., its magnitude is ...............degrees and the angle is an ................ [acute/obtuse] angle.

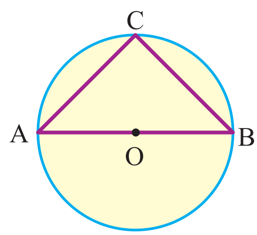
d) Observe that on the major arc ACB, central angle is ............. of inscribed angle or inscribed angle is ............. of central angle.

e) So we can say: ..................................... ............................................. ........

## Angle on a semicircle

We have learned about the central and inscribed angles on the same arc of a circle. We also learned about the relationship between these two angles. Now let's try to know about the semicircular angle of a circle and what can be the measure of this angle.

## Manual task – 1



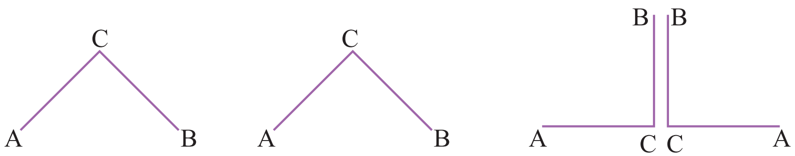

Step – 1: Draw a circle on your notebook using compass. Mark the center of the circle as O.

Step – 2: Draw the diameter AB through point O. The diameter divides the circle into ....... equal arcs. Now take any point C on any arc.

Step – 3: Join A, C and B, C. Hence ∠ACB is created which is the semicircular angle.

Step – 4: Now cut out two ∠ACB with the help of tracing paper. Then place the angles side by side as shown in the picture below.

Step – 5: What did you understand? The two angles are supplementary and equal, right?




Since, $\angle ACA = 180°$, $\therefore \angle ACB = \frac{1}{2} \times 180° = 90°$

**Pair task:** Discuss with a classmate and do the following:

a) Draw several circles of different radii.

b) Draw multiple semicircular angles on any semicircle of each circle and mark them.

c) Measure the semicircular angles in each case and write their degree measurements in a notebook.

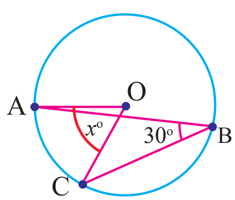
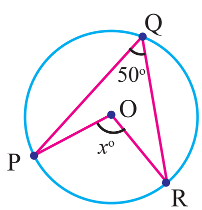
d) Write the conclusion that can be drawn by observing the degree measurement of the semicircular angles.

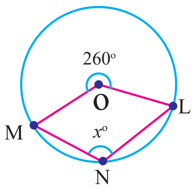
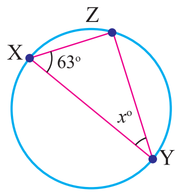
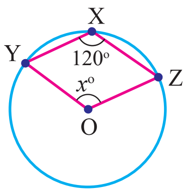
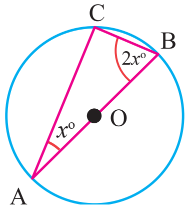
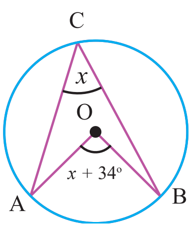
**Conclusion:**

..........................................................................................................

## Individual task:

Determine the value of $x$ by solving the following problems with reasoning based on your experience and observations. In each case consider O to be the center of the circle.

| Figure for the problem | Solution |
|---|---|
| a)  | a) |
| b)  | b) |

| | |
|---|---|
|   c) | c) |
|   d) | d) |
|   e) | e) |
|   f) | f) |
|   g) | g) |

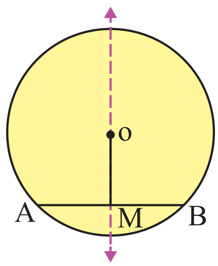
## A game of circle and chords

Let's try to learn about the relationships that can be formed between the circle, the center of the circle and chords of different lengths.

## Manual task– 2

Step – 1: Draw a circle of any radius on the paper with a pencil-compass. Mark the center of the circle as O and cut off the circular area.

Step – 2: Draw a chord AB which is not a diameter.




Step – 3: Fold the circular area in such a way that the fold passes through the point O and one part of the line segment AB lies on the other.

Step – 4: The fold intersects the line AB at a point M. Add O and M.

Step – 5: Measure ∠AMO and ∠BMO with the help of protractor. What do you think? Are the two angles equal? Do the two angles measure 90°? That is ∠AMO = ∠BMO= a right angle.

Step – 6: Measure the length of AM and BM using a centimeter scale. What do you see? AM = BM right?

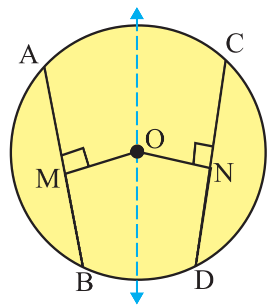
Now draw multiple circles with radius of different lengths and repeat the steps several times. Every time ∠AMO = ∠BMO= is a right angle and AM = BM. right? Then we can say –

**A perpendicular drawn from the center of the circle to a chord which is not a diameter bisects that chord at right angle.**

**Individual task:** Cut out some paper and do the task manually:

If a straight line passes through the center of the circle bisects a chord which is not a diameter of the circle, then the straight line will be perpendicular to that chord.

## Manual task– 3:




Step – 1: Draw a circle of any radius on your notebook with a pencil-compass. Mark the center of the circle as O.

Step – 2: Draw two chords AB and CD of equal length in the circle.

Step – 3: Now from the center O draw perpendiculars OM and ON on chords AB and CD respectively. Find out if there is any relationship between these two perpendiculars.

Step – 4: Draw the diagram on a tracing paper and cut out the circle.

Step – 5: Fold the circle in such a way that point A coincides with point C and point B coincides with point D.

Step – 6: Observe whether point M falls on point N? Must have fallen, right? Then unfold the tracing paper and you will see that the fold passes through the center point O. So, you know from here that OM = ON. If you want, you can check it by measuring it with a scale.

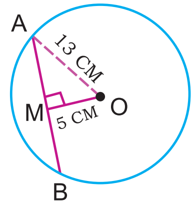
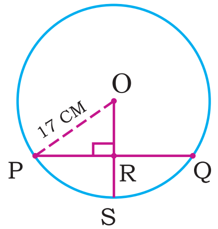
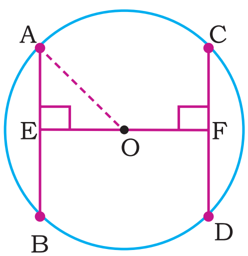
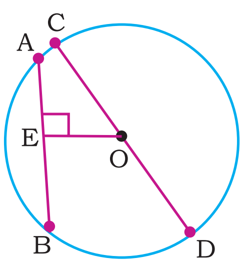
Now draw multiple circles with radii of different lengths and repeat the steps several times. In every case, you get OM = ON, right? Then we can decide –

**All equal chords of a circle are equidistant from the center**

**Individual task:** Now you know after working manually, **"All equal chords of a circle are equidistant from the center"**. But is the opposite possible? That is, if two chords are equidistant from the center of the circle, will the length of the two chords be equal? Verify by doing the work manually.

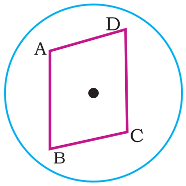
## Individual task:

Solve the following problems with reasoning based on your experience and observations. In each case consider O to be the center of the circle.

| **Figure for the problem** | | **Solution** |
|---|---|---|
| a) Determine the length of the chord AB. |  | a) |
| b) If PQ=30 cm, RS = ? |  | b) |
| c) If AB = CD, EF = 6 cm and BE = 4 cm, OA = ? |  | c) |
| d) If CD = 26 cm and OE = 10 cm , AB = ? |  | d) |

## Cyclic Quadrilateral

You have already made many models during the circle and stick activity by putting one or two sticks of different lengths between circular rings of various sizes. At the same time, you got to know various new information. Now using the pencil-compass and scale, draw some cyclic quadrilaterals on the notebook. How to draw? Quadrilaterals can be easily drawn by drawing a few circles of different radii and marking four points A, B, C and D on each circle and adding the points in order.




Is the quadrilateral ABCD in the figure cyclic? Explain with logic.

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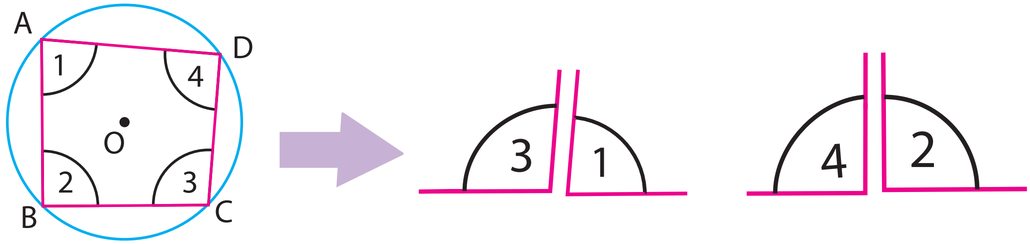
Now measure the angles of the circular quadrilaterals drawn in the notebook and complete the table 7.2 below:

**Table 7.2**

| Figure | ∠A | ∠B | ∠C | ∠D | ∠A + ∠C | ∠B + ∠D |
|---|---|---|---|---|---|---|
| 1. | | | | | | |
| 2. | | | | | | |
| 3. | | | | | | |
| Decision after observing the table: | | | | | | |

## Manual task – 4:




Step – 1: Draw a circle of any radius on the notebook with a pencil-compass. Mark the center of the circle as O.

Step – 2: Take four points A, B, C and D on the circle. Construct quadrilateral ABCD by joining A, B; B, C; C, D and D, A.

Step – 3: Cut out the circular area and mark the angles of the quadrilateral ABCD with numbers 1,2,3,4.

Step – 4: Cut the angles carefully and separate them.

Step – 5: Now place the opposite corners side by side as shown in the figure.

Step – 6: What do you obtain? $\angle 1 + \angle 3 =$........... and $\angle 2 + \angle 4 =$...........

### Individual task:

Verify by doing this manually:

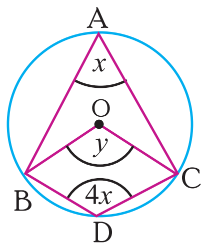
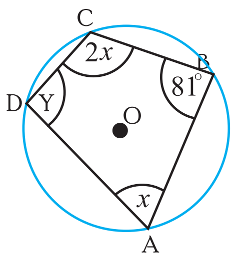
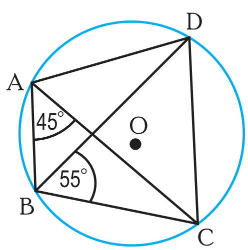
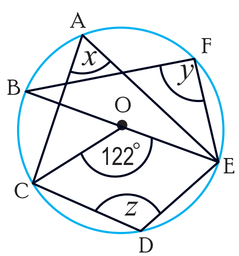
If the sum of the opposite angles of any quadrilateral is 180° or two right angles, the vertices of the quadrilateral are concylic.

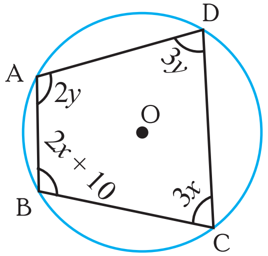
**Concyclic:** If the vertices of a bounded area inscribed in a circle lie on the circumference of that circle, then those points are called concyclic.

- Draw a some concyclic polygons in the notebook and explain your logic.

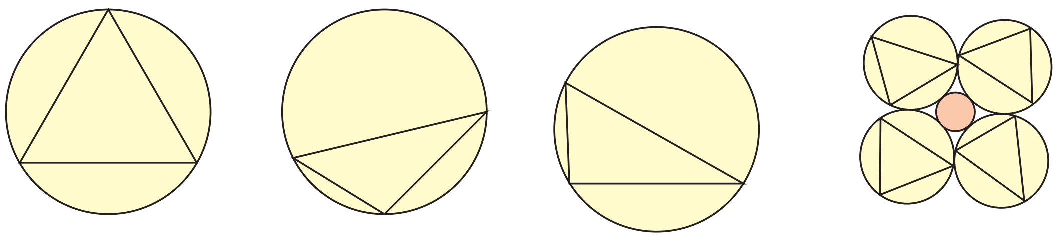
Write in the blank space below what you have learned doing the manual task:

**Individual task:** Brainstorm and solve the following problems. In each case consider O to be the center of the circle.

| Figure of the problems | Solutions |
|---|---|
| a)   Determine the values of $x$ and $y$. | a) |
| b) Determine the values of $x$ and $y$.   | b) |
| c)   If ∠DBC= 55° and ∠BAC=45°, ∠BCD=? | c) |
| d)   Determine the values of $x$, $y$ and $z$. | d) |

| | |
|---|---|
| e)   Determine the values of $x$ and $y$. | e) |

## Circumcircle of a Triangle

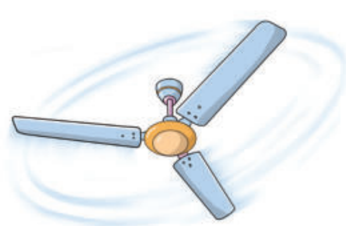


You notice floor designs painted on national days and various social occasions. If you notice, you will see that there are several designs painted on your handkerchiefs, table covers, bed sheets, etc. These designs are basically different types of geometric shapes. Dipa draws several circles on the notebook with radii different lengths. Then makes a triangle inside each surrounding circle to form the design as shown above, where the vertices of each triangle are on the circle. Can you tell what the circle drawn like this and the triangle inside the circle are called?

Since the circle circumscribes the triangle contained within the circle, the circle is the **circumcircle** of the triangle.

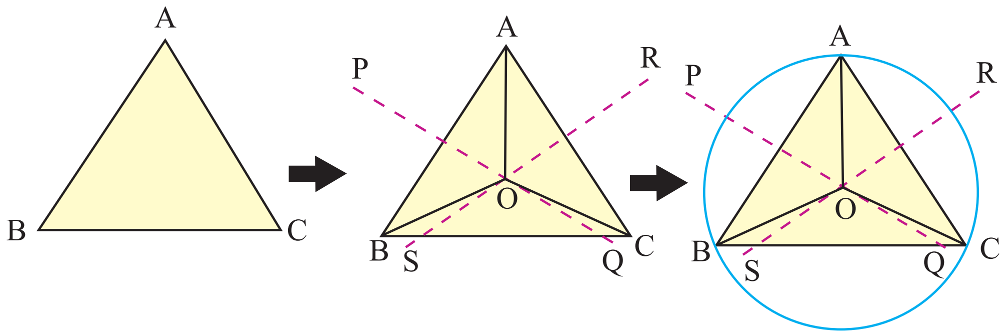
The following example will help you understand more about the circumcircle of a triangle.

When a fan hanging from the ceiling above your head in the classroom or at home rotates, a ................ geometric shape is created. Now, in your imagination, you tie the open ends of the fan with each other using a thin rope or thread. It forms a triangle. Now if the fan rotates, its circle will be the circumcircle of the triangle.



As you must understand from the discussion, an inscribed triangle can be easily found by adding any three points on any circle. But given a triangle of any shape, how to draw the circumcircle of that triangle? To solve the problem, let's try to manually draw the circumcircle of a triangle of any shape:

## Manual task– 5




Step – 1: Draw any triangle ABC on notebook or any white paper. Then cut out the triangular area from the notebook or other paper.

Step – 2: Now fold the side AB of triangle ABC in such a way that point A coincides with point B. Now open the fold and mark the perpendicular bisector PQ by drawing a line along the fold.

Step – 3: Find the perpendicular bisector RS of side AC by folding similarly.

Step – 4: Note that the perpendicular bisectors PQ and RS intersect at a point. Denote the point of intersection by O. Measure with a scale and see that the distance from point O to points A, B and C will be equal. That is, OA = OB = OC.

Step – 5: Now draw a circle through point O with radius equal to OA or OB or OC. What do you see? The circle passes through the vertices A, B and C of ΔABC. right?

Think about what we can call the point O?

Point O can be called **circumcentre** of ΔABC and the circle you get centered at point O is the **circumcircle** of ΔABC and OA or OB or OC is the **circumradius** of ΔABC.

## Individual task

a) Draw an obtuse-angled and a right-angled triangle and draw the circumcircles of the two triangles.

b) In the table below, state the positions of the circumcenters of acute-angled, obtuse-angled and right-angled triangles by drawing figures.

| | **Acute-angled triangle** | **Obtuse-angled triangle** | **Right-angled triangle** |
|---|---|---|---|
| Circumcircle | | | |
| Position of the circumcenter | Inside the triangle | | |

c) Lengths of sides of a triangle are 9 cm, 12 cm and 15 cm.

(i) Find the length of the circumradius of the triangle.

(ii) Find the area of the circumcircle of the triangle.

## Incircle of a Triangle

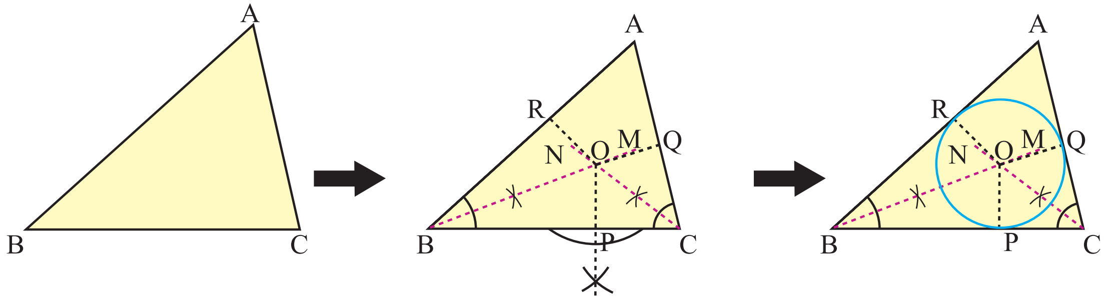
Ahana draws a triangle on her notebook. She wants to identify a point inside the triangle from which the sides of the triangle are always equidistant.

### Task – 1:

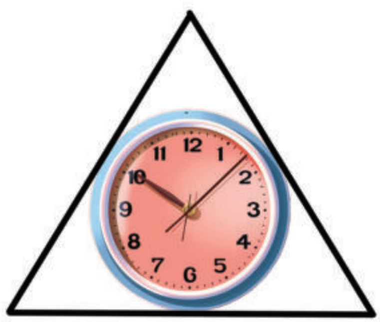
First she cuts out the triangle. Then she draws the perpendicular bisector of each side of the triangle. The perpendicular bisectors drawn by Ahana meet at a point. Now she measures the distance of each side of the triangle from the obtained point with a scale and notices that the distances are not equal. So, she thinks of alternatives and does the following accordingly.

### Task – 2:

- Ahana draws another triangle ABC in her notebook and cuts out the triangle.
- Now to get the bisector of ∠ABC, she folds ∠ABC along the vertex A in such a way that the side AB coincides with the side BC.




- She unfolds the paper and draws the bisector BM of ∠ABC along the fold.
- Similarly, by folding the paper she finds the bisector CN of ∠ACB. It is observed that the bisectors of ∠ABC and ∠ACB of ΔABC intersect at a point. Ahana denotes the point of intersection by O.
- From point O, perpendiculars OP, OQ, OR are drawn on sides BC, AC, AB respectively. Ahana finds OP = OQ = OR by measuring with scale.
- Ahana drawa a circle with center O and radius equal to OP. It is seen that the circle also passes through points Q and R. That is, she draws a circle that touches all three sides of the triangle.



Ahana is very happy to draw the figure. Because the wall clock in her reading room is very much like her figure. What can we call the circle drawn by Ahana? Since the circle lies inside the triangle and touches all the three sides of the triangle, we can call it the **incircle** of the triangle. The center of the incircle is called **incentre** and the radius is called **inradius**.

## Individual task:

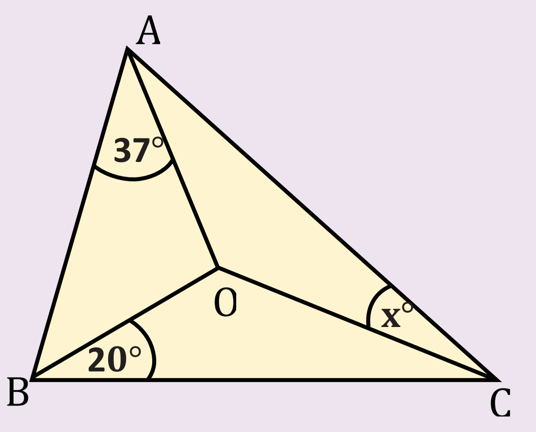
a) Draw an obtuse-angled and a right-angled triangle and draw the incircles of the two triangles.

b) In the table below, state the positions of the incenters of acute-angled, obtuse-angled, and right-angled triangles by drawing figures.

| | **Acute-angled triangle** | **Obtuse-angled triangle** | **Right-angled triangle** |
|---|---|---|---|
| Incircle | | | |
| Position of the incenter | | | Inside the triangle |

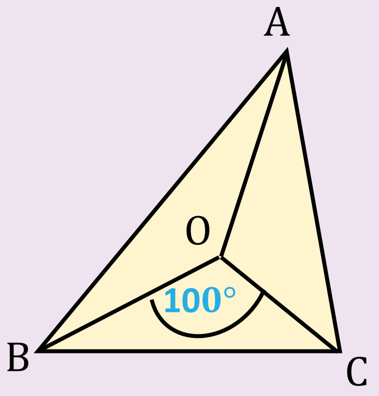
c) Construct an equilateral trinagle and check the positions of the circumcenter and incenter by drawing them manually.

d)




If O is the incenter of ΔABC, find out the value of $x$.

d)

| e) | e) |
|---|---|
| <br>If O is the incenter of ΔABC and ∠BOC = 100°, what is the value of ∠BAC? | |

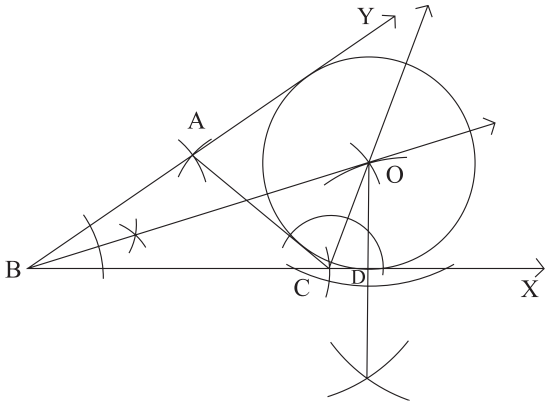
## Excircle of a triangle

We learned to draw circumcircles and incircles of triangles, one of which passes through three vertices of the triangle and the other is inside the triangle but touches all three sides. Can you draw a circle that is outside the triangle but touches all three sides of the triangle? That is, the circle will touch one side of the triangle and the extensions of the other two sides.

Let's try to draw the circle:

First draw any triangle ABC. Extend the sides BC and BA of ΔABC to X and Y.

You already know how to bisect an angle, right?




Now bisect the interior ∠ABC and the exterior ∠ACX. Notice that the bisectors intersect at a point. Denote the point of intersection by O.

Now from point O draw a perpendicular on the side AC and on the extension of sides BC and BA. Measure the length of three perpendiculars drawn from point O with a scale. The perpendicular drawn from point

O to the extension of side BC is OD. Now draw a circle with center O and radius equal to OD. The circle touches the side AC of ΔABC and the extensions of the sides BC and BA.

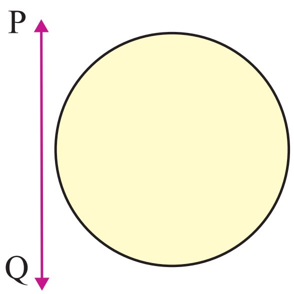
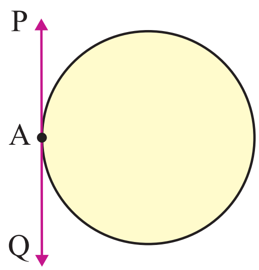
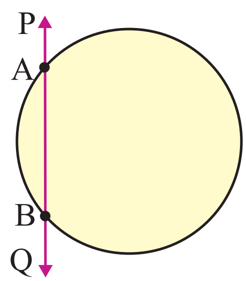
What is this type of circle called?

Although the circle lies outside the triangle, it touches one side of the triangle and the extensions of the other two sides. So, we can call this kind of circle **excircle** of triangle. The center of the circle is called **excentre** and the radius is called **exradius.**

Can you think and tell about how many excircles of a triangle can be drawn?

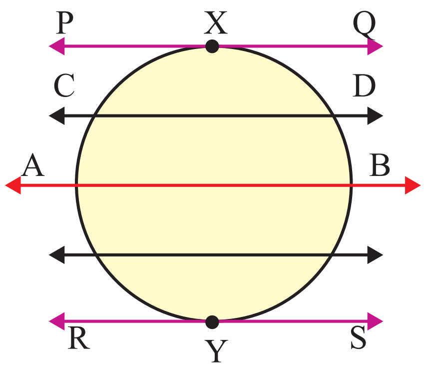
## Secant and Tangent of a Circle

Consider the relative positions of a circle and a straight line in a plane. Observe the table 7.3 below to see what positions the circle and the straight line can take:

| **Table 7.3** | | |
|---|---|---|
|  |  |  |
| a) The line does not touch the circle. | a) The line touches the circle at point A. | a) The line intersects the circle at points A and B. |
| b) There is no common point between the straight line and the circle. | b) There is one common point between the straight line and the circle. | b) There are two common points between the straight line and the circle. |

| | | |
|---|---|---|
| c) Circle and straight line are two different geometric shapes. There is no relation between them. | c) The straight line is a **tangent** to the circle and A is the **point of contact.** A tangent touches the circle at only one point. | c) The straight line is a secant of the circle and a **secant** intersects a circle in two points. |

If you want, you can make a tangent to the circle by hand. For this first draw a circle of any radius in the notebook. Then cut out the circular area. Now place a ruler on the circular area and draw two straight lines AB and CD by both sides of the ruler. Then the secant CD will be parallel to the secant AB. Now draw PQ and RS parallel to the secant AB by drawing other parallel secants with the help of ruler, which touch the circle at two points X and Y respectively. In this case both PQ and RS will be two tangents to the circle.

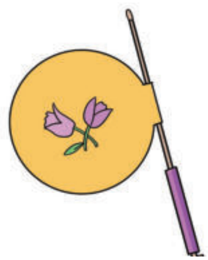



Use circular rings or old CDs to make toys like the one in the picture. What to call the handle of the toy?

## Where do we see tangents in our daily lives?

1. Can you tell the name of the object shown in the next picture? For those who don't know, a couple of clues can be given.
   i. If for some reason there is no electricity on a hot day, you fan yourself with it by hand.
   ii. Wind is generated by holding its handle and turning it left and right.
   iii. You can buy it from a shop or fair or make it yourself.

Wow! You are right. It is a hand fan. What is the round part of the fan and the handle called? As the handle is clamped by touching the circular area on the outside of the circular region. Hence the handle can be called tangent.



2. When you ride a bike on the road, the wheels of the bike keep rolling on the road. And the road will be a tangent to the wheel. Again, since the road touches both the wheels of the bicycle at the same time, we can call the tangent (which is the road) as common tangent. Since the centers of both wheels are on the same side of the road, the road can be called a direct common tangent.

## Group Work

If the centers of two circles are on opposite sides of the common tangent, then what can we call that common tangent?

Discuss with your classmates and write in your own notebook with reasons.

## Properties of Tangent

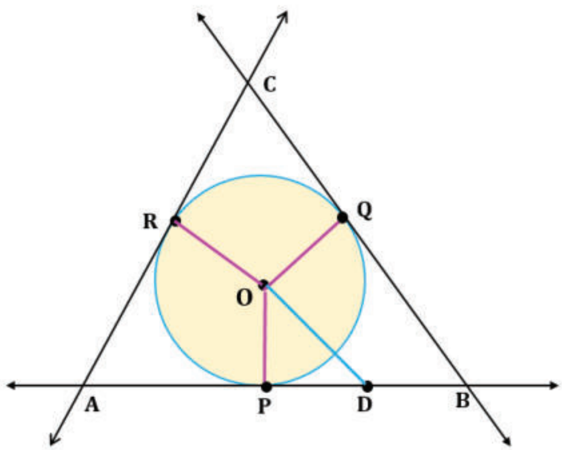
We already know that a tangent to a circle touches the circle at only one point. Now let's find out manually what other properties of the tangent have:

### Manual task– 6:

Step – 1: Cut out one-fourth of any page in the notebook. Draw a circle of any radius on the piece of paper.

Step – 2: Now take any three points P, Q and R on the circle.

Step – 3: Fold the paper and draw three tangents AB, BC and CA at points P, Q and R respectively as shown in the figure.




Step – 4: Add O, P; O, Q and O, R. What part of the circle did you get?

Step – 5: Now take any point D other than P on AB. Add O, D. Measure the length of OD and OP with a scale. What do you get? OD > OP, right?

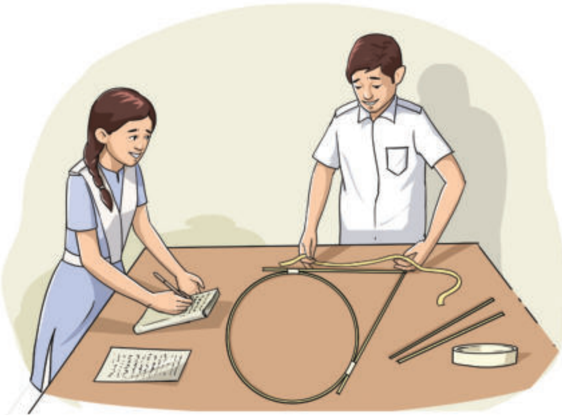
Then it appears that OP is the smallest of the straight lines joining the center and any point on the tangent AB. Measure ∠OPB and ∠OPA using a protractor. What did you get? Here $\angle OPB = \angle OPA = 90°$. Similarly measure the angles $\angle OQB$ and $\angle OQC$ and $\angle ORC$ and $\angle ORA$ for tangents BC and CA.

So, you can now decide, $OP \perp AB$.

**That is, the tangent drawn at any point of the circle is perpendicular to the radius passing through the point of contact.**

## Manual task – 7:

Another property of tangents is that if two tangents are drawn from a point outside the circle to the circle, the distances between the outside point with each the point of contact is equal.



Let's check manually:

The checking process requires a round ring, a few thin straight sticks, tape and a long scale.

Step – 1: Place the ring on the table and tape the two sticks to either side of the ring as shown.

Step – 2: Now tie the two open ends of the sticks together. It looks like two tangents from a point outside the two rings when the stick is attached to the circular ring doesn't it?

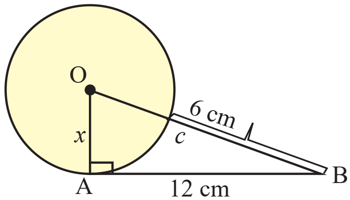
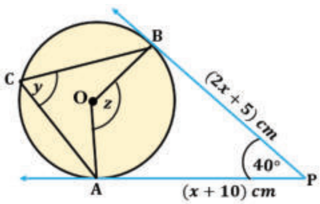
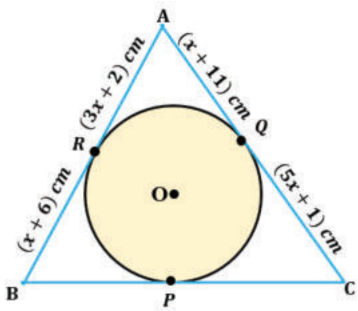
Step – 3: Measure the distance from the point where the two sticks are tied together to the point where they touch the circular ring.

What did you get? Are the two distances equal?

Repeat the process several times with two or three more round bangles of smaller or larger radius and sticks of different lengths. If you get the same result in all cases, you can now decide that- **if you draw two tangents to the circle from a point outside the circle, the distance from that outside point to the two tangent points will be equal in all cases.**

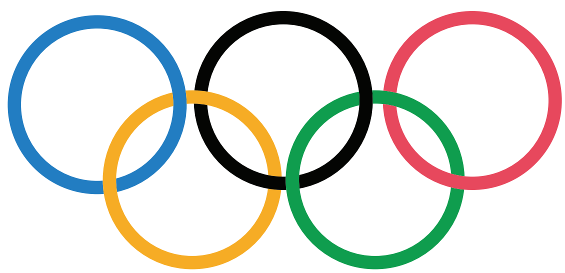
## Individual task:

Solve the following problems with reasoning based on your experience and observations. In each case consider O to be the center of the circle.

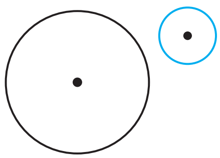
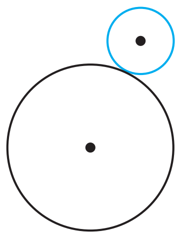
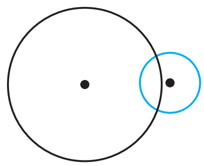
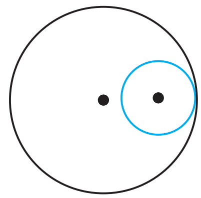
| Table 7.4 | |
|---|---|
| **Figure for the problem** | **Solution** |
| a)   Calculate the value of $x$. | a) |
| b)   Determine the values of $x$, $y$, $z$. | b) |
| c)   Determine the length of BC. | c) |

## Game of multiple circles and sticks

The next image is a very familiar logo. What can you tell us about the logo? Can multiple circles be chained like this using a pencil-compass? Discuss with your classmates and try to draw in the notebook.

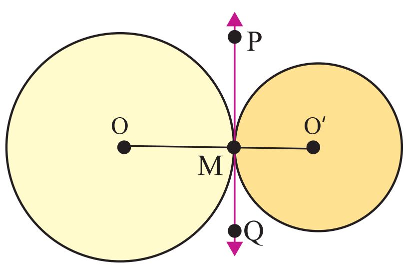


Suppose you are given two circular rings or bangles of different radii. You need to place the two rings or bangles on the notebook and draw a circle. The condition is that the two circles will touch each other at a point. That is, they will have a common point of contact. Ahana quickly drew a couple of circles like the picture below with two peaks. Observe Ahana's drawings closely. In which image do two circles have a common point of contact? Mark (√) on the correct figure and (×) on the wrong figure. Your answer must be supported by written logic.

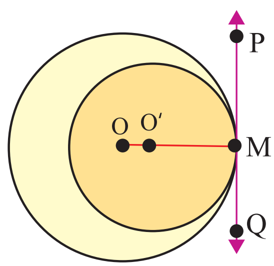
| **Table 7.5** | | | | |
|---|---|---|---|---|
| Figure | a)  | b)  | c)  | d)  |
| Correct/ Wrong | | | | |
| Logic | | | | |

Now look at the two imaxges on the side:

The points of contact of both Fig – A and Fig – B are same. Moreover, the point of contact and the centers of both circles lie on the same straight line. Think and fill the blanks:




চিত্র- ক




চিত্র- খ

The distance between the centers of the two circles in Fig. A is equal to ……… of the radii of the circles. The distance between the centers of the two circles in Fig. B is equal to ……… of the radii of the circles.

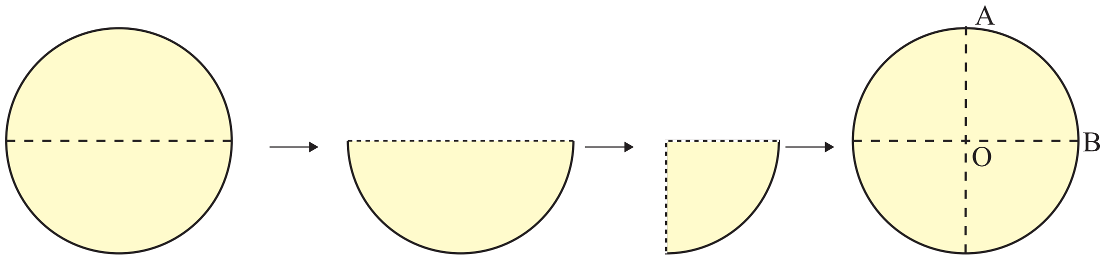
## Individual task:

You have several circular rings or bangles of different radii and several sticks of different lengths. Make three models using circular rings or bangles in such a way that the rings touch each other externally in the first, internally in the second and do not touch in the third. If necessary, the two rings can be tied with masking tape. Now construct common tangents of both types using sticks of different lengths on the models. Show and explain to the teacher by making the model containing the common tangent.

**Measuring length of arc, area of circular segment and sector**

Do you remember wanting to make a shelf in the corner of your study room? Your shelf is not a regular geometric shape. That is, all parts of the shelf are not equal. In some places it will need round wood, and in some places it will need wood like segments and sectors. That's why you need to understand these things. So let us now try to know about the length of an arc, how it is determined.

## Measuring the length of an arc




As you have learned in the previous class, a circle of radius r units has a circumference of $2\pi r$ units and an area of $\pi r^2$ square units. Using these information you will be able to find the length of the arc of a circle of radius *r* units.

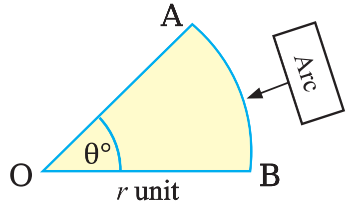
If a circular paper is folded into four equal folds, four equal sectors are formed, right?

You know that a circle has an angle of 360° at the center. Since you have folded the circular paper into four equal folds, the sector AOB will make a 90° angle at the center. And in this case the length of arc AB will be $\frac{1}{4} \times 2\pi r = \frac{\pi r}{2}$ units. But if the circular paper is folded in any way and not evenly folded, you cannot tell without measuring the degree of angle the sector will make at the center.

Let, the sector makes an angle $\theta°$ at the centre. In that case let us try to find out how to find the length of the arc of that circle.

You already know that, the length of the arc is directly proportional to the angle created at the center.

Hence, we can say, $\dfrac{\text{Length of the arc}}{\text{Circumference of the circle}} = \dfrac{\theta}{360}$



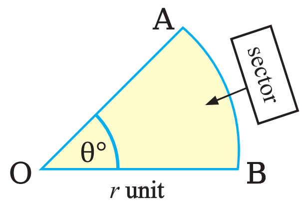

$\therefore$ Length of the arc $= \dfrac{\theta}{360} \times$ Circumference of the circle $= \dfrac{\theta}{360} \times 2\pi r$ sq. units.

## Measuring the area of a sector:

When you folded the circular paper into four equal folds, the sector AOB made a 90° angle at the center. And in this case the area of the sector AOB will be $\frac{1}{4} \times \pi r^2$ square units. But if the circular paper is folded in any way and not evenly folded, you cannot tell without measuring the degree of angle the sector will make at the center. Let, the sector makes an angle θ° at the centre. In that case let us try to find out how to find the area of the sector of that circle.

You already know that, the area of the sector is directly proportional to the angle created at the center.

Hence, we can say, $\dfrac{\text{Area of the sector}}{\text{Area of the circle}} = \dfrac{\theta}{360}$

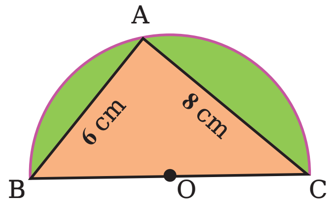
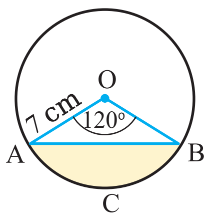



**$\therefore$ Area of the sector $= \dfrac{\theta}{360} \times$ Area of the circle $= \dfrac{\theta}{360} \times \pi r^2$ units.**

| Problem | Solution |
|---|---|
| 1. An arc creates an angle of 30° at the centre. If the radius of the circle is 12 cm, find the length of the arc. | 1. Given that, radius of the circle, $r$ =12 cm, and the angle created at the center by the arc, θ =30°. <br> $\therefore$ Length of the arc $= \dfrac{\theta}{360} \times 2\pi r$ units <br> $= \dfrac{30}{360} \times 2 \times 3.1416 \times 12$ cm $= 6.28$ cm (approx.) |

| | |
|---|---|
| 2. A sector creates an angle of 60° at the center. If the radius of the circle is 8 cm, find the area of the sector. | 1. Given that, radius of the circle, $r = 8$ cm, and the angle created at the center by the sector, $\theta = 60°$.<br>$\therefore$ Area of the sector $= \frac{\theta}{360} \times \pi r^2$ sq. units<br>$= \frac{30}{360} \times 3.1416 \times 6^2$ sq. cm $= 18.85$ sq. cm (approx.) |

## Individual Task

| Problem | Solution |
|---|---|
| 1.<br><br><br>If ABC is a semi-circle, what is the area of the green portion in the figure? | |
| 2.<br><br><br>O is the center of the circle. Determine the area of the shaded circular section ACB. | |

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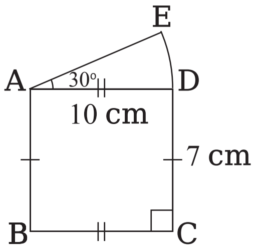
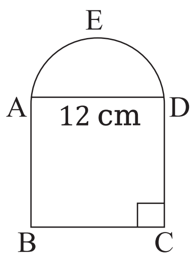
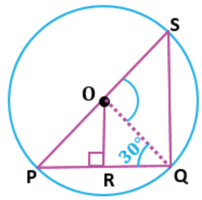
<table>
<tr><td>3.<br><br>In the given figure, ABCD is a rectangle. DAE is a circular segment. $\angle DAE = 30°$<br><br>Determine the area of the whole region.</td><td></td></tr>
<tr><td>4.<br><br>In the given figure, ABCD is a square. DAE is a semi-circle. Determine the area of the whole region.</td><td></td></tr>
</table>

# Exercise

1. O is the center of the circle. The chord PQ = $x$ cm and OR $\perp$ PQ.



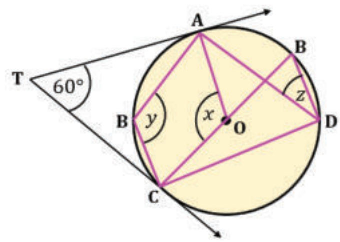

   a) What is the measure of $\angle QOS$?

   b) If OR = $\left(\frac{x}{2} - 2\right)$cm, determine the value of $x$.

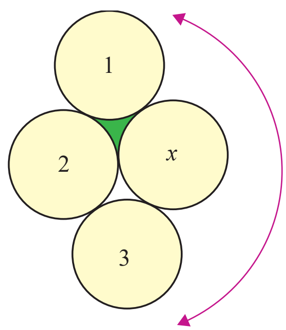
2. Parallel chords PQ and RS of length 10 cm and 24 cm are on opposite sides of the center of the circle whose center is at O. If the distance between the chords PQ and RS is 17cm, calculate the radius of the circle.

3. Suppose you have a triangular piece of land. The perimeter of the land is 124 meters. You want to cultivate vegetables in the maximum area of that land. If the perimeter of the vegetable garden is 84 m, find the area of the plot.

4. In the figure, O is the center of the circle and TA and TC are two tangents. If $\angle ATC = 60°$, determine the values of the angles $x$, $y$ and $z$.




5. Collect several one (1) tk. coins of the same size (of the same type). Place any one of the coins in the center of your notebook. Now place the coins touching each other around it as shown in the picture. It's like arranging pieces on a carrom board.




   a) Touching in the way shown in the figure, what is the maximum number of coins that can be placed around the coin marked '$x$'? Solve the diagram by completing it.

   b) Join the centers of the three circles marked '1', '2' and '$x$' in the given figure. Suppose the perimeter of the triangle obtained is 18 cm. Using this information, find the area of the green part of the figure.

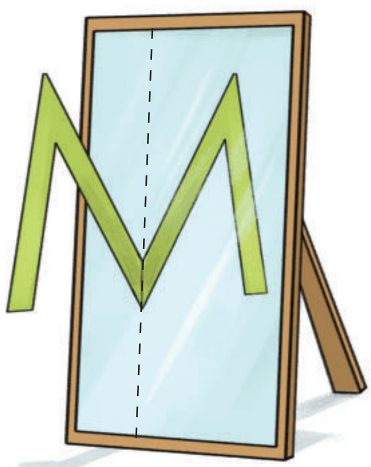
   c) Using any of the coins draw a circle on your notebook. Then find the center of the circle.

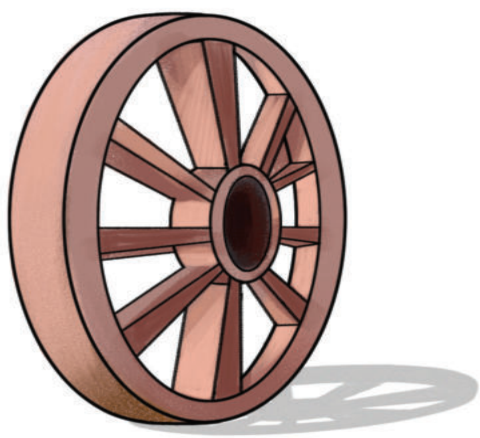
   d) Draw two circles whose radii are multiples of the radius of any coin. If two circles are tangent to each other on the outside, prove that the distance between the centers of the two circles is twice their average radius.

# Application of Symmetry in Measurement

## You can learn from this experience

- Symmetrical object and Symmetry
- Symmetrical line
- Test of Symmetry
- Rotational Symmetry
- Characteristics of Rotational Symmetry





# Application of Symmetry in Measurement

What if we could measure an object by measuring a part of it ? How can we identify these objects from our surrounding? Carefully observe the objects shown below:

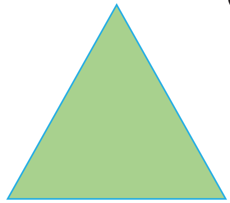

(a)

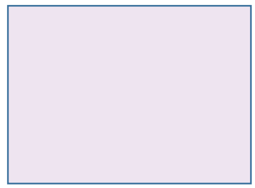

(b)


(c)

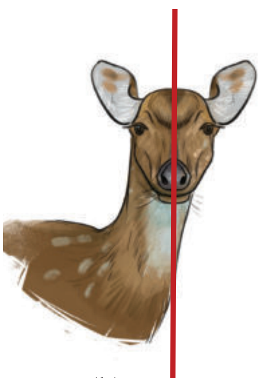

(d)

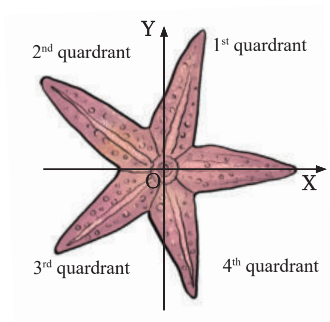

(e)


(f)

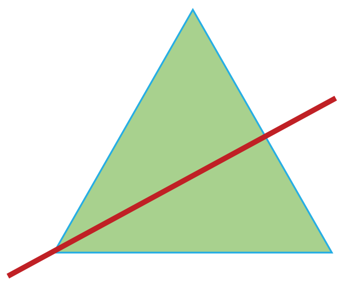

(g)

Now think if the objects above can be classified in a way that if we measure a part of the object we can easily measure the whole. Let me give you some examples:


(h)



(i)


(j)

The vertical line has divided the face of the deer into two equal halves. Notice that one line has divided the triangle into two equal parts. In the same way the horizontal line has divided the star fish in the equal halves. Then notice that a line has divided the triangle into two equal parts also. In this way we can easily measure the whole from one of the divided parts.

If we divide an object in the middle and one portion matches with the other fully/ completely we can call it Symmetrical object with characters of symmetry. Here the line that parts them equally is called the Line of Symmetry

## Individual Task

Do the following task and match it with your classmate. If there is any errors take reasonable decision after discussing.

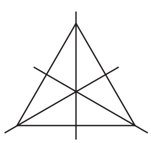
1. How many symmetrical lines are there in the flower? Show all the symmetrical lines drawing a flower.
2. Draw all the symmetrical lines of the equilateral triangle.
3. Draw and show all the symmetrical lines of the rectangle.

It is not possible to find out all the symmetrical lines and symmetry of object by drawing and folding paper. Now let us find it through a pair work.

## Pair work

Make these figure by cutting out paper. Match it by folding the paper and fill in the blank space after discussing with your partner.

**Table 8.1**

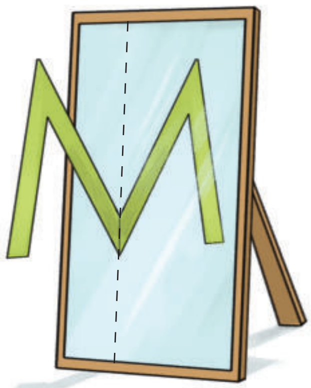
| Figure | Matches after folding in the middle/Does not match | Number of Symmetrical lines |
|---|---|---|
| 1.Equilateral Triangle  | Match | 3 |
| 2.Rectangle | | |
| 3.Scalene Triangle | | |
| 4.Isosceles Triangle | Does not match | |
| 5. Equilateral Triangle | | |
| 6. English alphabet T | | |
| 7. English alphabet L | | |

## Individual task

Name 5 symmetrical objects and draw their image. Draw the lines of symmetry of these objects.

## Let us examine Symmetry with a Mirror

We can do another activity to understand Symmetry. We can use a mirror. First make a symmetrical structure by cutting paper. Suppose You have made a shape like English alphabet ‘M’. Cut the letter in such a way so that one part of M reflected in the mirror like the complete letter “M”as shown in the right side picture.

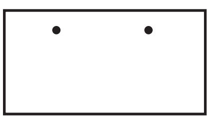


examine Symmetry with a Mirror

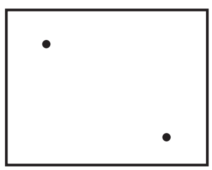
What can you see? Certainly you can see the complete letter “M”. Here the line along which you cut the letter “M” is its line of symmetry. In this way we can identify Line of symmetry by reflection. Hence line of Symmetry is also called Reflectional symmetry.

## Individual task:

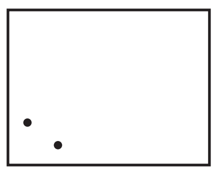
Draw Line of Symmetry of following diagram.



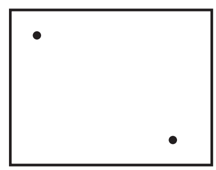
(1)



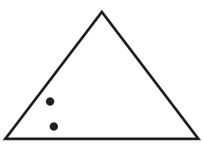
(2)



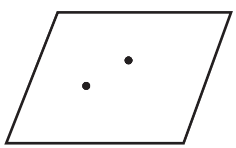
(3)



(4)



(5)

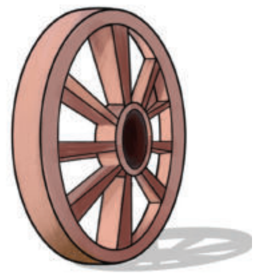


(6)

(7)

We have seen Symmetry of Line . Now , think if it is possible to show any other object Symmetrical.

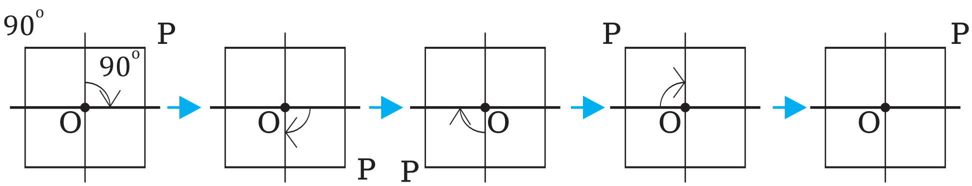
Notice the picture of the wheel. If you rotate the wheel by 40 degree for once will it look the same ? Will there be any change in its size and shape due to the rotation? Here change has occurred in different parts of the wheel.



Here if the wheel is rotated 9 times ($40° \times 9 = 360°$) again it will come back to its initial condition. It means the wheel has rotational symmetry. Here rotational angle is 40 degree and the degree of symmetry is 9.

Suppose you have drawn a Square. Now rotate the square clockwise by 90 degree.

During rotation observe after how many times (of 90 degree rotation) it has returned to initial position?




So , we have seen that after rotating the square at a specific angle it looks like the same.

And after rotating for specific times, it comes back to the previous position. The object rotates around a particular point. A fixed point around which a symmetrical object rotates is called centre of rotation. An object can be rotated both clockwise and anti clockwise. Here no change occurs in centre of rotation, angles of rotation, degree of rotational symmetry. Only the direction of rotation changes. Now we can say, The object which has Rotational Symmetry has four features .

1. Rotational angle 2. Degree of rotational symmetry 3.Centre of rotation 4. Direction of rotation

Draw and identify the Angle of rotation, Degree of rotational symmetry, Centre of rotation and Direction of rotation of the Square given above

## Individual Task:

Draw picture in your exercise book and fill in the blank space.

| Table 8.2 | | |
|---|---|---|
| **Figure** | **Rotation angle** | **Degree of Rotational Symmetry** |
| 1. Square | | |
| 2. Equilateral Triangle | | |
| 2. Balanced hexagon | | |
| 4. Isosceles Triangle | | |
| 5. Balanced Pentagon | | |
| 6. English alphabet T | | |
| 7. | | |
| 8. | | |
| 9. | | |

There are many objects around you that have Line Symmetry and Rotational Symmetry.

Observe and identify those objects. Write down their names and reasons for selecting these.

## Group Work

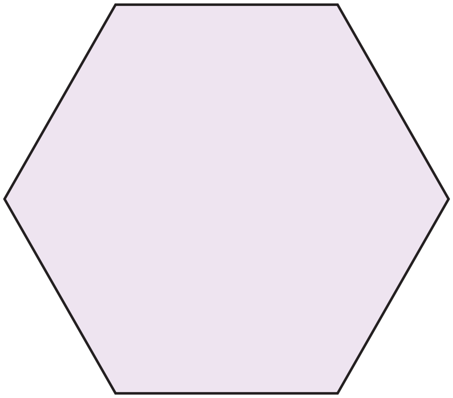
Discuss with the members and fill in the blank space.

**Table 8.3**

| Figure | Line Symmetry | Number of symmetrical line | Rotational Symmetry | Rotation angle | Degree of Rotational symmetry |
|---|---|---|---|---|---|
| Square | | | | | 4 |
| | | | | | |
| | | | | | |
| English Alphabet H | | | Yes | | |
| | | | | | |
| | | | | | |
| English Alphabet Z | No | | | | |
| Circle | Yes | | | | infinite |

## Let us arrange the garden using Symmetry

A regular polygon shaped model of land for garden is shown in the picture.

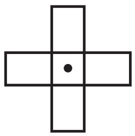
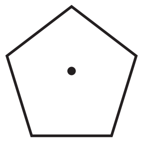
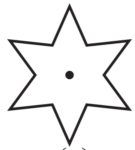
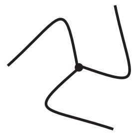
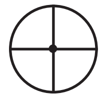
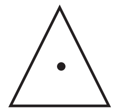


The length of each side is 10meter. You have to arrange the garden with different kinds of flower plants. Everyone of you will get the same size of land.

- Divide the land into equal six portions. Suppose your group has got one portion. What is the size of that?
- What is the total land area of the garden?
- Suppose you are given tk500 to make the garden. Everyone will arrange their garden buying trees with that amount. How much will it cost to decorate the garden?
- Explain how you have got the total land area
- How many parts the garden can be divided into? Give reasons for your answer.

# Exercise

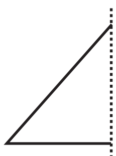
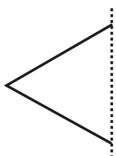
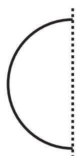
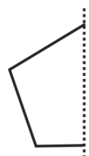
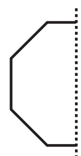
1. Find the degree of Rotational Symmetry and Rotational angle of the following Figure


(a)

(b)

(c)

(d)

(e)

(f)

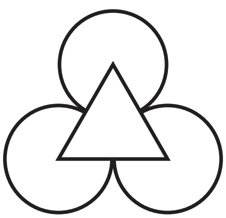
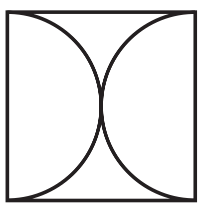
2 a) What does one degree srotational symmetry mean? What is the angle of one degree rotational symmetry?

b) Can there be 20 degree Symmetrical angle?

3. The Line of Symmetry is given. Complete the Figure.


(a)

(b)

(c)

(d)

(e)

4.Draw the Line of symmetry of the following Figure.


(a)

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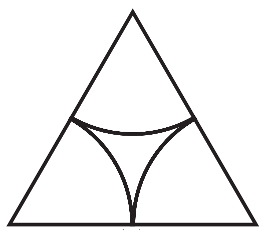
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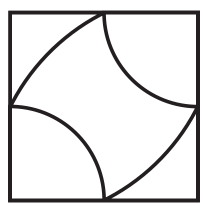
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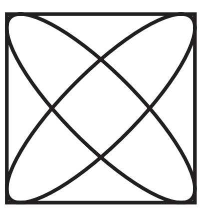
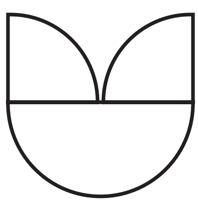
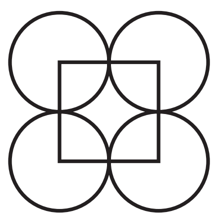
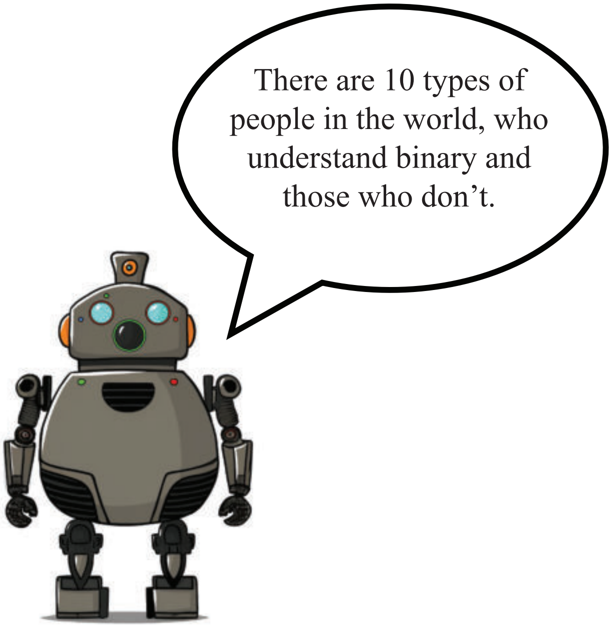

(e)

(f)

(g)

# The Binary Number System

## You can learn from this experience

- Clear concept of base of Binary number System
- Necessities and uses of binary numbers
- Transformation between Decimal and Binary number system
- Operations of Binary numbers



# The Binary Number System

In class 7, you worked with binary numbers. You might have asked, why are we learning a number system based on 2? After all, the calculations we do are based on 10, and we solve everything using the decimal number system! Before we explore this question, let's see how the binary system we got.

The German mathematician Gottfried Wilhelm Leibniz  was the proponent of the binary number system. One of his topics of interest was how to transform the language of religious philosophy into mathematical logic. From this thought, he tried to express all the numbers that could be expressed by ten decimal digits, only by 0 and 1. Furthermore, he showed that the arithmetic (addition, subtraction, multiplication, division) that can be done with the decimal digits, also works for the system with 0 and 1. These developments were published in his book "Explanation of the Binary Arithmetic" in the year 1703. 150 years later, an Irish school teacher named George Boole wrote in his book "The Mathematical Analysis of Logic" that, what happens in our daily life is some combination of truths and falsities, which we can express with 0 and 1. But Boole's statement was an algebraic expression of Leibniz's theory, which was a groundbreaking development that linked the presence and absence of electricity in modern computers to mathematical logic.

Gottfried Wilhelm Leibniz

Binary arithmetic in Leibniz's handwriting

You will learn how computers are built using Boolean logic in your Digital Technology classes and also in later grades. But before that, you have to learn Leibnitz's binary system. Earlier, you were given an idea about the structure of the binary system.

Let me know, how were the experiences about Binary number system.

## Quiz

1. What is the full form of Bit?
2. Why does the binary system use two digits only? Explain.
3. Express the binary number 1011 in decimal form.
4. What will be the number if we express 11 from Binary to Decimal?


You learned binary quite well, isn't it? Now, let me tell you why we learn binary.

You might have worked on a computer or played games on it. The work you did or the games you played can be saved on the computer, right? When we find a letter or a book, we usually keep it on the table or put it in a drawer. The good memories we have, are saved in our brains. So, where does the computer save your work? Write down your thoughts about this in the space below.

Our dependency on computers is increasing day by day. If we do not understand how computers work, we can not use them completely. This machine works based on the binary number system. Just like we add/subtract numbers now and then, so does a computer, but in the binary system. So, if we know how to add/subtract/multiply/divide in binary, we will have an idea of how a computer works.

## Base

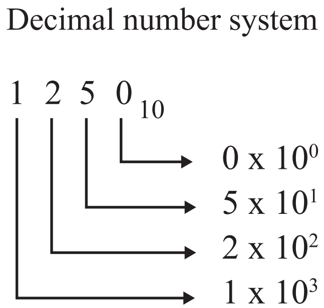
Consider the decimal system. It is made up of 10 digits: 0 to 9. So its base is 10. In the decimal system, 250 is expressed as $(250)_{10}$.

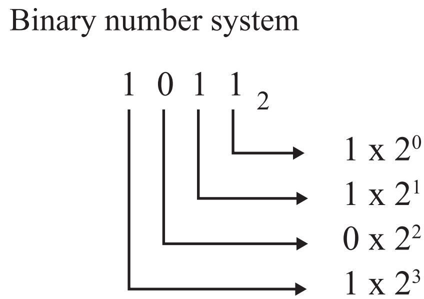
On the other hand, the binary system is made up of 2 digits: 0 & 1. So its base is 2. In binary, 1011 is expressed as

$(1011)_2$

## Place Value

You have learnt place value of different digits of a number in decimal number system. Here we will learn place value of different digits of a number in Binary number system. We have discussed comparing them in the following.

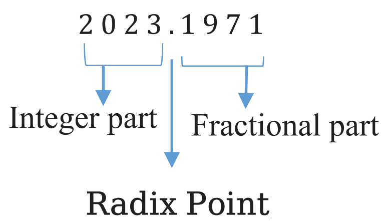



**Individual task:** Write place value of each digit of the Binary number $(11011)_2$

## Radix Point

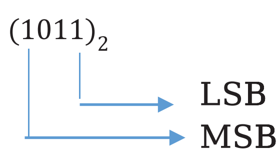
A number can have two parts. The integer part and the fractional part. The integer part and the fractional part are separated by a radix point. For example-




## Significant Digit

The digit with the highest place value is called the Most Significant digit. The digit with the lowest place value is called the Least Significant digit. In Binary system digit is called Bit.So, in Binary system, Most Significant Bit is called **MSB** and Least Significant Bit is called **LSB.**

### Example:



## Uses of 0 and 1 in Digital machine

Since Binary system is used in machine and machine can only detect the presence and absence of electricity. So, mathematicians have assigned '0' for no electricity and '1' for the presence of electricity.

## Conversion

We humans count using the decimal number system. So it's a part of our human language. But electronic devices can't detect anything but binary signals, so binary is the language of machines (Machine Language). Humans created Machine Language, machines did not. However, when we have to instruct computers to do something, we have to translate our language into machine language.

## Decimal to binary

Divide the integer part of the decimal number by 2, while noting the quotient and remainder. Then divide the quotient again by 2, noting the new quotient and remainder. We continue dividing the quotients by 2 until we get a quotient of 0. Finally, we write the remainders in reverse order, this is the integer part of the binary number.

**Example:** Convert $(23.25)_{10}$ to Binary

**Solution:**

a. $(23)_{10} = (?)_2$

| | | | |
|---|---|---|---|
| 2 | 23 | Remainder | |
| 2 | 11 | 1 | LSB ↑ |
| 2 | 5 | 1 | |
| 2 | 2 | 1 | |
| 2 | 1 | 0 | |
| | 0 | 1 | MSB |

$\therefore (23)_{10} = (10111)_2$

b. $(0.25)_{10} = (?)_2$

| | | |
|---|---|---|
| | | .25 |
| | | × 2 |
| MSB ↓ | 0 | .5 |
| | | × 2 |
| LSB | 1 | .0 |

$\therefore (0.25)_{10} = (.01)_2$

$\therefore (23.25)_{10} = (10111.01)_2$

## An alternative method of conversion

We know, every bit has a fixed place value. Let's convert $(23)_{10}$ into binary. The smallest place value equal to or closest to 23 is 16. Let's list the binary place values up to 16. Now, write 1 over 16 as shown. This means we have one 16.

| 1 | | | | | |
|---|---|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 | → Place values |

To have the decimal number 23, we need a 7. Adding 4, 2 and 1 gives 7. So we write 1 over 4, 2 and 1 each. As for the place values we did not use, we write 0 over them.

| 1 | 0 | 1 | 1 | 1 | |
|---|---|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 | → Place values |

$\therefore (23)_{10} = (10111)_2$

## Binary to decimal

Multiplying the bits with their respective place values and taking the sum of the products gives us the expected decimal number. For example,

$(101101)_2 = (?)_{10}$

1 0 1 1 0 1

$1 \times 2^0 = 1$

$0 \times 2^1 = 0$

$1 \times 2^2 = 4$

$1 \times 2^3 = 8$

$0 \times 2^4 = 0$

$1 \times 2^5 = 32$

$(45)_{10}$

There is another simple method. Write down the place value under the bits. For the bits that have 1, we add the place values. This sum is the required answer. For example,

| 1 | 0 | 1 | 1 | 0 | 1 | |
|---|---|---|---|---|---|---|
| 32 | 16 | 8 | 4 | 2 | 1 | → Place values |

$32 + 8 + 4 + 1 = 45$

$\therefore (101101)_2 = (45)_{10}$

## Binary number processing

In the meantime, you have learnt about the concept of Binary number system. Here We will learn practically, how addition, subtraction, multiplication and division of binary numbers are processed.

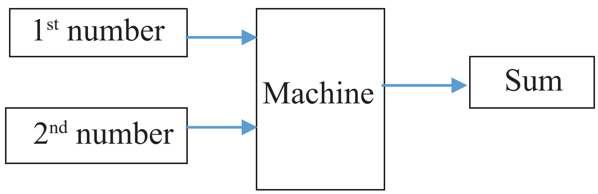
## Addition of Binary numbers

Suppose you are told to add the decimal numbers 2 and 3. This is what you do:

$$2 + 3 = 5$$

What if you have a machine, where you enter two numbers and their sum comes out?

For example,




But it is hard to explain decimal number in machine. You have to talk with it in the binary language. We have only two digits in binary. So, we can determine all possible binary sums in the following table.

| Table of addition of Binary digit | | | | |
|---|---|---|---|---|
| 0 | + | 0 | = | 0 |
| 0 | + | 1 | = | 1 |
| 1 | + | 0 | = | 1 |
| 1 | + | 1 | = | 0<br>(1 in hand) |

Notice that if you perform the fourth operation in decimal, then the result is 1 + 1 = 2.

Write the decimal number 2 in binary form:

How many digits are required to express $2_{10}$ in binary? Ans:

The value of 2 of Decimal system is 10 in Binary system. You have to write 0 keeping 1 in hand.

Now, let's use the method we have just learned to do a binary addition. For your convenience, the decimal addition is also shown alongside.

## Example 1

| Binary | Decimal |
|---|---|
| 1 0 1 1 | 1 1 |
| (+) 1 0 1 | (+) 5 |
| 1 0 0 0 0 | 1 6 |

Here, we see that we can easily add to Binary numbers as decimal numbers. Evaluate the following binary sums. If needed, convert to decimal to verify your work.

(1)

$$\begin{array}{r} 101 \\ (+)\ 11 \\ \hline \\ \hline \end{array}$$

(2)

$$\begin{array}{r} 1101 \\ (+)\ 111 \\ \hline \\ \hline \end{array}$$

(3)

$$\begin{array}{r} 1111 \\ (+)\ 1000 \\ \hline \\ \hline \end{array}$$

(4)

$$\begin{array}{r} 1011 \\ (+)\ 101 \\ \hline \\ \hline \end{array}$$

(5)

$$\begin{array}{r} 10101 \\ (+)\ 1010 \\ \hline \\ \hline \end{array}$$

(6)

$$\begin{array}{r} 100001 \\ (+)\ 11110 \\ \hline \\ \hline \end{array}$$

But, decimal numbers have fractional parts. For example,

$$\begin{array}{r} 29.31 \\ (+)\ 5.05 \\ \hline \\ \hline \end{array}$$

How do we perform the addition above? Work it out and write the answer in the space below. Also, explain how you have used the radix point while doing the addition.

You will be glad to know that you can sum fractions of Binary in the same way like as decimal system. Let's do a sum.

### Example 2

| Binary | Decimal |
|---|---|
| 1101.101 | 13.625 |
| (+) 1011.011 | (+) 11.375 |
| 11001.000 | 25.000 |

Now do quickly some binary additions and verify by converting to decimals.

(7)

110.101
(+) 110.001

________________

________________

(8)

111.111
(+) 10.101

________________

________________

(9)

1011.10110
(+) 110.01101

________________

________________

**Choose any one of the following:**

☐ a) Direct addition in binary is easy. No need to verify using decimals.

☐ b) Converting to decimals and then converting the result into binary is easier.

## Binary Subtraction

We can do subtraction of Binary numbers like as decimal numbers. You have been doing subtraction of decimal numbers for many years. To realize complexity, do the following two subtraction of decimal numbers.

**Problem 1.**

$$\begin{array}{r} 10 \\ (-)\ 4 \\ \hline \\ \hline \end{array}$$

Explain your steps in doing the subtraction.

**Problem 2.**

$$\begin{array}{r} 1008 \\ (-)\ 994 \\ \hline \\ \hline \end{array}$$

Write with explain the procedure of how you do this. Is it complex than previous one? Where is the complexity? Write it.

You must observe that there is a fact to borrow or to take hand. Observe the following example.

Subtraction of two numbers in decimal system with borrowing

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (Borrowing row) | 0 | 9 | 9 | 9 | 13 | 14 | 10 | |
| | 1 | 0 | 0 | 0 | 4 | 5 | 0 | 1 |
| (−) | 0 | 0 | 8 | 0 | 5 | 7 | 3 | 0 |
| | 0 | 9 | 1 | 9 | 8 | 7 | 7 | 1 |

If lower digit is greater than the upper digit at the time of subtraction,then you have to add a tens with top digit borrowing from the left of top digit For this, one tens is reduced from left digit. In the previous example, 0 is above the subtrahend number of tens 3. Here 5 is the left digit of 0. Adding a tens (10) with 0 from 5 you can get 10, which is put above 0. On the other hand, 5 is now 4 subtracted by 1. Now, since the digit of hundreds of subtrahend 7 is greater than 4, so taking a tens from left digit you can get 14. This rule has been used for other digit also.

The above rule may be not familiar with you. However, if you practise, you will be familiar with this method.

## Pair work

Subtract in the following using borrowing method

1. 50083 – 9354 2. 15703 – 15691

What if this method did not exist? Let's learn a method of subtraction without the borrowing or taking in hand method. For this, We have to learn about complement number.

## Complement number of decimal number

Can you tell me, what number will be added with 40 to get 99. You will surely say, 59. Here 59 is complement number of 40. On the other hand, 40 is complement number of 59. That is, 40 and 59 are complement with each other relative to 99. Again, 40 and 959 are complement with each other relative to 999. In decimal number system, this type of complement is called 9's complement. 9's complement of any number $a$ is denoted by $a^*$. $a^* + 1$ is called 10's complement of $a$. 10's complement of any number $a$ is denoted by $a^{**}$. That is, $a^{**} = a^* + 1$.

**Example:** Determine 9's complement and 10's complement of 6,54 and 104 relative to 999.

**Solution:**

Let $a = 54$. then relative to 999,

9's complement of $a$ is $a^* = 999 - 54 = 945$

10's complement of $a$ is $a^{**} = 945 + 1 = 946$

Do yourself 9's complement and 10's complement of 6 and 104.

Now, We will subtract in decimal number system without borrowing or, without taking in hand. Here, We will use the concept of 9's complement and 10's complement.

**Example:** Subtract 365 from 3064 using concept of complement.

**Solution:**

Since, 3064 is a number of 4 digit. So, to solve this you have to find out the complement of 365 relative to 9999.

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$$3064 - 365 = 3064 + \underbrace{9999 - 365}_{9's \text{ complement}} - 9999$$

$$= 3064 + \underbrace{9634 + 1}_{10's \text{ complement}} - 9999 - 1$$

$$= 3064 + 9635 - 10000$$

$$= 12699 - 10000$$

$$= 2699$$

We can subtract Binary numbers like as subtraction of decimal numbers. How many ways two Binary digits can be subtracted are given below.

| **Table of subtraction of Binary digit** | | | | |
|---|---|---|---|---|
| 0 | – | 0 | = | 0 |
| 0 | – | 1 | = | 1, Borrowing 1 |
| 1 | – | 0 | = | 1 |
| 1 | – | 1 | = | 0 |

Using this rule, do the following subtraction.

## Individual task

Convert binary number to decimal number and subtract in both both the system of the following to verify it.

| Binary | | Decimal |
|---|---|---|
| 1011 | → | |
| (–) 101 | → | |
| | → | |

## Brain Storming

- Explain your steps in doing the subtraction.

- Did you commit any mistake? Did you repeat any step?

Do the following subtractions and verify your work.

(10)

$$\begin{array}{r} 110 \\ (-)\ 110 \\ \hline \\ \hline \end{array}$$

(11)

$$\begin{array}{r} 111 \\ (-)\ 101 \\ \hline \\ \hline \end{array}$$

(12)

$$\begin{array}{r} 101110 \\ (-)\ 11001 \\ \hline \\ \hline \end{array}$$

(13)

$$\begin{array}{r} 10110 \\ (-)\ 11001101 \\ \hline \\ \hline \end{array}$$

The subtraction of binary fractions is the same as the subtraction of decimal fractions. Do the following subtractions and verify your work.

(14)

$$\begin{array}{r} 110.101 \\ (-)\ 110.001 \\ \hline \\ \hline \end{array}$$

(15)

$$\begin{array}{r} 111.111 \\ (-)\ 10.101 \\ \hline \\ \hline \end{array}$$

(16)

$$\begin{array}{r} 1011.10110 \\ (-)\ 110.01101 \\ \hline \\ \hline \end{array}$$

(17)

$$\begin{array}{r} 1011.10110 \\ (-)\ 110.01101 \\ \hline \\ \hline \end{array}$$

We can also subtract Binary numbers like as subtraction of decimal numbers using borrowing method. Observe the following example.

Subtraction of two numbers in binary system with borrowing

$$\begin{array}{rccccccc} \text{(Borrowing row)} & 0 & 1 & 10 & 1 & 1 & 10 & 10 \\ & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ (-) & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ \hline & 0 & 1 & 1 & 1 & 0 & 1 & 1 \end{array}$$

## Pair work

Subtract using borrowing method of the following Binary numbers

1. 10011 – 1001 2. 110111 – 10001

We can easily subtract of Binary numbers comparing with the complement system of decimal numbers.

## Complement of Binary numbers

1's complement of any Binary number $a$ is denoted by $a^*$ like as decimal number. And 2's complement of $a$ is denoted by $a^{**}$. That is, $a^{**} = a^* + 1$.

**Example:** Determine 1's complement and 2's complement of Binary number 101101.

**Solution:** Let $a = 101101$. then,

1's complement of a is $a^* = 111111 - 101101 = 010010$

2's complement of a is $a^{**} = 010010 + 1 = 010011$

## Individual Task

Determine 1's complement and 2's complement of Binary numbers of the following

(i) 1011 (ii) 1100 (iii) 10001

Now, We will subtract in Binary number system without borrowing or without taking in hand. Here, We will use the concept of 1's complement and 2's complement.

## Example:

$$100011 - 101 = 100011 + \underbrace{111111 - 101}_{1's\text{ complement}} - 111111$$

$$= 100011 + \underbrace{111010 + 1}_{10's\text{ complement}} - 111111 - 1$$

$$= 100011 + 111011 - 1000000$$

$$= 1011110 - 1000000$$

$$= 11110$$

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## Pair work:

Subtract the followings using the concept of complement number.

(i) 1011 – 101 **(ii)** 101001 – 100110 **(iii)** 1110101 – 100011

Write in the following blank space if you have any question at the time of doing the mathematical operations of above.

## Binary Multiplication

With much hard work you learnt binary addition, subtraction. Now we will learn multiplication and division. Binary multiplication is very easy and is similar to decimal multiplication.

| **Table of multiplication of Binary digit** | | | | |
|---|---|---|---|---|
| **0** | × | **0** | = | **0** |
| **0** | × | **1** | = | **0** |
| **1** | × | **0** | = | **0** |
| **1** | × | **1** | = | **1** |

So let's see how it's done.

**Example:** $(1011)_2 \times (101)_2 = (?)_2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 0 | 1 | 1 | | 11 |
| | | (×) | 1 | 0 | 1 | | (×) 5 |
| | | 1 | 0 | 1 | 1 | | |
| | 0 | 0 | 0 | 0 | × | | |
| 1 | 0 | 1 | 1 | × | × | | |
| 1 | 1 | 0 | 1 | 1 | 1 | | 55 |

Now, do some on your own.

(18)

$$\begin{array}{r} 1101 \\ (\times)\ 111 \\ \hline \\ \hline \end{array}$$

(19)

$$\begin{array}{r} 101110 \\ (\times)\ 11001 \\ \hline \\ \hline \end{array}$$

(20)

$$\begin{array}{r} 100001 \\ (\times)\ 11110 \\ \hline \\ \hline \end{array}$$

(21)

$$\begin{array}{r} 111.111 \\ (\times)\ 10.101 \\ \hline \\ \hline \end{array}$$

## Binary Division

By now we know how to add, subtract and multiply binary numbers. The division of binary numbers follows some rules. As in the decimal system, division by 0 is undefined in the binary system too. The rules of binary division are:

| **Table of division of Binary digit** | | | | |
|---|---|---|---|---|
| 0 | ÷ | 0 | = | Undefined |
| 0 | ÷ | 1 | = | 0 |
| 1 | ÷ | 0 | = | Undefined |
| 1 | ÷ | 1 | = | 1 |

These two rules applied together allow us to divide binary numbers just like decimal numbers.

Let's see an example.

$$\begin{array}{r} 1011)\ 110111\ (101 \\ -\ 1011\phantom{00} \\ \hline 1011 \\ -\ 1011 \\ \hline 0 \end{array}$$

## Pair work 1

Divide using division method of the following problems of binary numbers.

1. 1010 ÷ 10 2. 111011 ÷ 1011 3. 10111010 ÷ 1001

## Pair work 2

Below given are some divisions with decimals. Convert the numbers into binary and then divide.

1. 100 ÷ 25 2. 77 ÷ 7 3. 85 ÷ 5 4. 128 ÷ 32

## Exercise

1. Convert the following Binary numbers to decimals.

   i) 010101 ii) 110011 iii) 100011 iv) 101000

   v) 101100 vi) 001100.101 vii) 010010.111 viii) 0010111111.11

2. Convert the following decimal numbers to Binary numbers.

   i) 6 ii) 19 iii) 56 iv) 129

   v) 127 vi) 96 vii) 25 viii) 200

3) Find the sum of the following numbers.

   i) 101111 + 101101 ii) 10101 + 100010 iii) 1010101 + 1000001

4. Find the sum of the following numbers converting decimal to Binary.

   i) 6 + 19   ii) 10 + 32   iii) 56 + 16   iv) 127 + 127

5. Find the subtraction of the following Binary numbers.

   i) 1001 – 101   ii) 11001 – 1011   iii) 1010010 – 111011

6. Determine 10's complement of the following decimal numbers.

   i) 2351   ii) 90152   iii) 10003   iv) 9999

7. Subtract using complement of the following decimal numbers.

   i) 43101 – 5032   ii) 70081 – 6919   iii) 2173901 – 5835

8. Determine 2's complement of the following binary numbers.

   i) 1111   ii) 1011001   iii) 1010101   iv) 1000001

9. Subtract using complement of the following binary numbers.

   i) 11001 – 1001   ii) 100101 – 10011   iii) 11000101 – 101101

10. Multiply converting decimal to Binary of the following decimal numbers.

    i) 18 × 6   ii) 32 × 23   iii) 21 × 7   iv) 59 × 18

    v) 118.2 × 46   vi) 180.50 × 65   vii) 192 × 22   viii) 111 × 101

11. Devide converting decimal to Binary of the following decimal numbers.

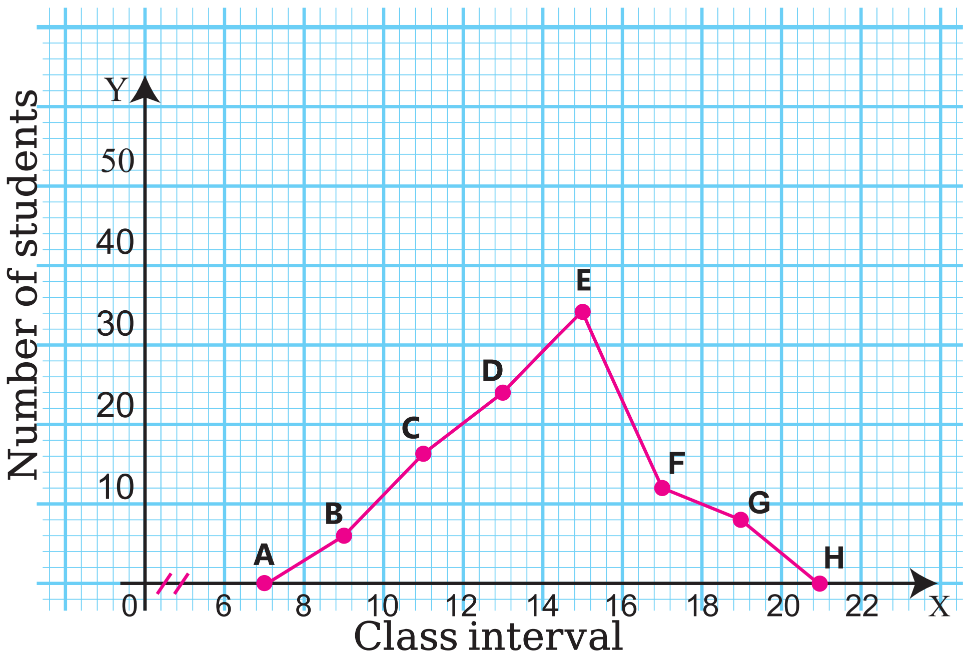
    i) 16 ÷ 4   ii) 34 ÷ 17   iii) 15 ÷ 3   iv) 99 ÷ 99

    v) 157 ÷ 46   vi) 180 ÷ 69   vii) 192 ÷ 22   viii) 111 ÷ 101

# Let us take decision after understanding information

## You can learn from this experience

- Data Processing
- Presenting data and information
- Frequency Polygon
- Cumulative Frequency Curve
- Measuring central tendency



# Let us take decision after understanding Information

In the previous grade we got basic conception regarding information and data . We also learned them in detail. In this experience we shall learn the type of information source, process of selecting right source, analyzing information. We shall also try to learn various ways of presenting information so that we can take effective measures to solve any problem. We need to identify how far we should rely on the source of information when we take decision on different issues in our everyday life. From the first experience of the Mathematics book you have learnt that for taking correct and effective decision it is very essential to collect data and information from reliable sources. This lesson is arranged in a way that you will take rational decision after collecting various important information through active participation in a project. You know that statistics is the branch of knowledge which helps us to take better and effective decisions through collecting and analyzing data and information. We hope you will be able to internalize the important skills by participating actively in this experience.

## Group Project

Let us start the task of the Project. First make Groups. Select any one from the given topics for collecting information

**Topics:**

1. Attendance of Grade VI and Grade VIII students
2. Present Condition of our Health based on Grade and age
3. Status of active members in your family
4. Details of the normal growth of trees in your garden

You will notice that if the information about the topics of the given list can be collected correctly/accurately then it becomes easier to take decision. For example, if we want to take decision for improving the quality of education of any school we must know the rate of attendance of the students. Let us do it altogether

Instruction for collecting Data: For your convenience the task of collecting data is mentioned along with the name of six sample groups. Collect the data in a planned way and preserve it. Teacher will inform you the time and give necessary instruction.

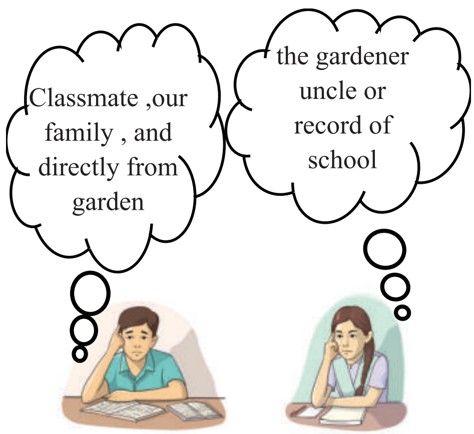
- Waterlily : Collect all the data of attendance of Grade VI students over the last one month
- Palash : Collect all the data of attendance of Grade VI students over the last one month

- Togar : Collect all the data of attendance of Grade VI students over the last one month
- Rose : Measure the height (in cm) and weight ( in kg) of all the students and keep record
- Shiuli : Collect number of members in the family and data of their age from all of the students of grade VIII
- Dahlia : Measure the height of the trees in the garden ( in cm or metre) and length of leaves ( in mm) and keep record

Following the time and instruction directed by the teacher for collecting data, first do the plan and distribute the responsibility among yourselves. The points you have noticed regarding data collection observe and write those down in your note book. Discuss with the other members of the group. Improve your plan if needed.

Now based on the data you collected in group prepare the given table in your exercise book and fill it in. Take opinion of all the members and help from the Teacher if required.

| Topic for collecting information | Probable sources for collecting information | Type of source | Which source is the most reliable? Why ? |
|---|---|---|---|
| | | | |
| | | | |
| | | | |




## Why is processing data and information necessary ?

The task after collecting data and information is to process information. But we can not start processing data immediately after collecting it, Because accurate data processing is not possible without arranging the collected data. You certainly remember you did the task of organizing data in the previous grade.

**Group Work:** Organize your collected raw data in a table

## Organizing the Data

We know that quantitative information are data of statistics. Data under research are the raw material of statistics. Most of the time these are in unorganized form. That is why organizing data is required. We know in order to table data

- First identifying the scope is needed
- Have to find Class number in right class interval
- Numerical information will be of one or the other class. Hence against the class, tally mark is used for numerical value for finding frequency
- The number of tally mark will show the frequency of the class and it should be written in the frequency column.

Suppose the data of attendance of Grade VI of your school are collected. Following the steps the unorganized data given below are organized. This activity will become easier if you follow the steps.

Let us organize the data and include it in the table:

Collected raw data of attendance of 100 students over the last one month after tabling:

18, 14, 8, 16, 9, 15, 13, 14, 15, 9, 17, 8, 15, 8, 10, 10, 11, 14, 16, 11, 10, 11, 18, 10, 11, 10, 12, 12, 13, 18, 12, 13, 12, 10, 12, 13, 12, 13, 11, 12, 13, 14, 11, 14, 15, 14, 15, 14, 14, 10, 14, 15, 14, 19, 15, 14, 17, 15, 14, 13, 15, 14, 16, 15, 15, 14, 15, 12, 17, 10, 16, 15, 12, 17, 15, 14, 10, 16, 9, 17, 13, 12, 16, 13, 11, 16, 12, 18, 13, 19, 15, 15, 19, 13, 12, 12, 14, 19, 14, 15

Here the highest value of data is 19 and lowest value is 8

$\therefore$ Range = (19 – 8) + 1 = 12;

You know,

$$\text{Number of classes} = \frac{\text{Range}}{\text{Class width}}$$

(converted into integer)

**Table -1**

| Class interval | Tally mark | Frequency |
|---|---|---|
| 8 – 10 | 𝍸 \| | 6 |
| 10 – 12 | 𝍸 𝍸 𝍸 \| | 16 |
| ………….. | ………….. | ………… |
| ………… | ………….. | ……….. |
| | Total | |

If we consider class width 2 ,

Then class number will be = $\frac{12}{2}$ = 6

Now let us organize in table the unorganized data of 100 students of Grade Nine. Two of them are done for you. Do the rest of them.

**Self reflection of Groupwork:**

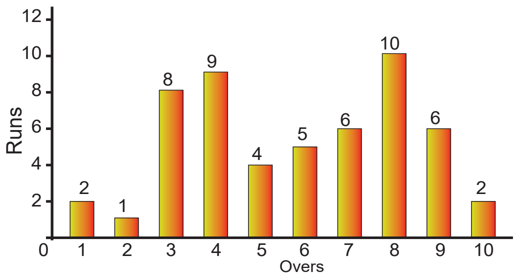
A list of all the tasks you di while doing group work is given. Arrange them according to sequence. Which of these tasks you found most challenging. Discuss in group and write it down.

Data organize → data collect → data classify → verifying reliability of source → finding out the range → selecting source → determining class interval

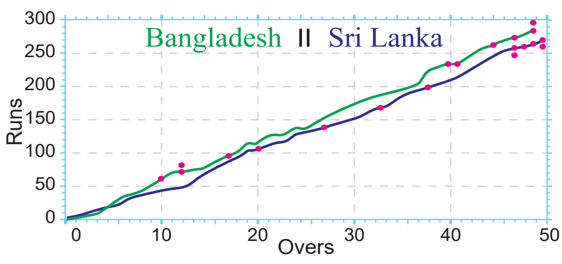
## Presentation of data

Present the collected data while showing your group work to the class in a way that everyone can easily understand the meaning of your data. Data can be presented in different ways.

Observe the following Figures.




(Figure A)




(Figure B)

Figure 10.1

We have seen this kind of presentation of data in television, magazine, dailies, advertisement, haven't we? You know that a picture/ an image is equals to thousand words. The information you can not express in a report of thousand words, sometime, one figure or image can express it completely. In the above bar chart and graph there are many data and information. Discuss and write down the data and information that you find in those two figures in the box given below.

We have learnt that when the collected data are organized and tabled it becomes easy to know about them and take decision. If the tabled data are presented through graph it becomes easy not only to understand and explain but also attractive and enjoyable. For this tabling unorganized data and presenting it through graph is much practiced and widely used system. In the previous grades detail discussion was done on bar graph, line graph, histogram, and pie chart. You also learnt how to draw them . Now let us discuss how to draw Frequency polygon and Cumulative Frequency Curve.

Now we shall try to draw Histogram using Table 10.2 .

**Table 10.2**

| Class interval | 8 – 10 | 10 – 12 | 12 – 14 | 14 – 16 | 16 – 18 | 18 – 20 |
|---|---|---|---|---|---|---|
| Number of students | 6 | 16 | 24 | 34 | 12 | 8 |

First, place the values of the class limits of the table along the *x*-axis (horizontal line) consecutively without any gaps. Here, take five times the length of side of the smallest square of the graph paper equal to 2 units. Since it is starting at 8, not the value 0, (-//-) is used to denote previous cells on the *x*-axis (horizontal line).

Now along the y- axis (vertical line), take five times the length of side of the smallest square of the graph paper equal to 10 units. Then draw some adjacent rectangles like the given figure. Their heights are equal to the frequencies and widths are equal to class intervals. In this way we graphically represent continuous data by a Histogram.

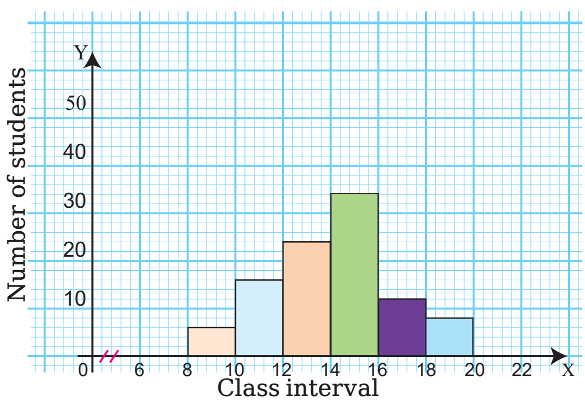
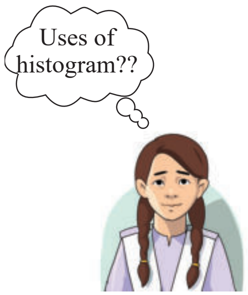



Figure 10.2




## Individual Task

Explain what is histogram to your friends (in your own word)

## Group Work

Draw histogram using the organized data

## Draw Frequency Polygon from histogram

| Table 10.3 | | |
|---|---|---|
| **Class** | **Mid point of class** | **Number of students** |
| 8 – 10 | 9 | 6 |
| 10 – 12 | 11 | 16 |
| 12 – 14 | 13 | 24 |
| 14 – 16 | 15 | 34 |
| 16 – 18 | 17 | 12 |
| 18 – 20 | 19 | 8 |

First determine the middle points of rectangle of histogram of attendance of 100 students.

Now mark the midpoints of the sides opposite and parallel to the x-axis as B, C, D, E, F and G. Here the *x*-coordinate of each point will be class mid-value and *y*-coordinate will be the frequency of the class. So the co-ordinates of the points B,C,D,E,F and G

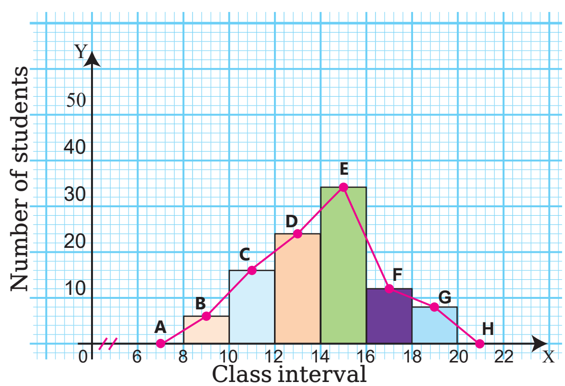



Figure 10.3

will be respectively (9, 6), (11, 16), (13, 24), (15, 34), (17, 12) and (19, 8). Now join the points using straight lines. Is the polygon complete? To complete the polygon, mark A(0, 7), which is the mid-value of the class interval which is right before the first class interval and mark H(21, 0), which is the mid-value of the class interval which is right after the last class interval. Now join A, B and G, H with straight lines. Then ABCDEFGH is our required polygon.

**Hence, the graph of the polygon obtained by joining the points expressing frequencies of classes with respect to mid-values of the class interval is called the frequency polygon.**

### Individual task:

a) Draw Frequency Polygon using organized data of the group. Every member of the group will do this in their exercise book

b) Think and prove area of polygon = area of histogram

## Draw Frequency Polygon without histogram

Drawing Frequency Polygon without histogram is also possible. Let us see how frequency polygon can be drawn using table 2 without histogram.

Here without drawing the rectangles, we mark the points B(9, 6), C(11, 16), D(13, 24), E(15, 34), F(17, 12) and G(19, 8) on the graph paper considering the class mid-values as $x$-coordinates and frequencies as $y$-coordinates.

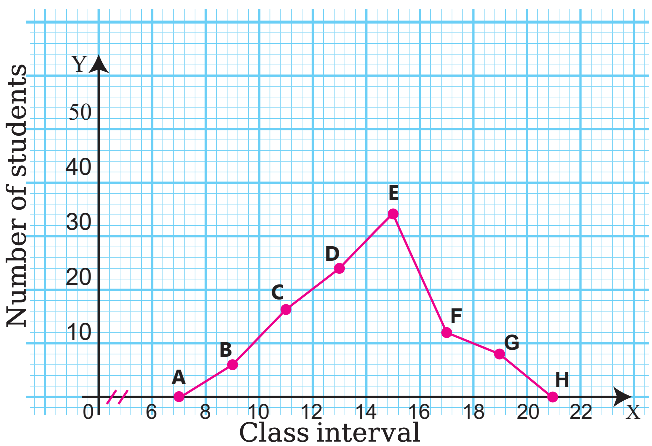



Figure 10.4

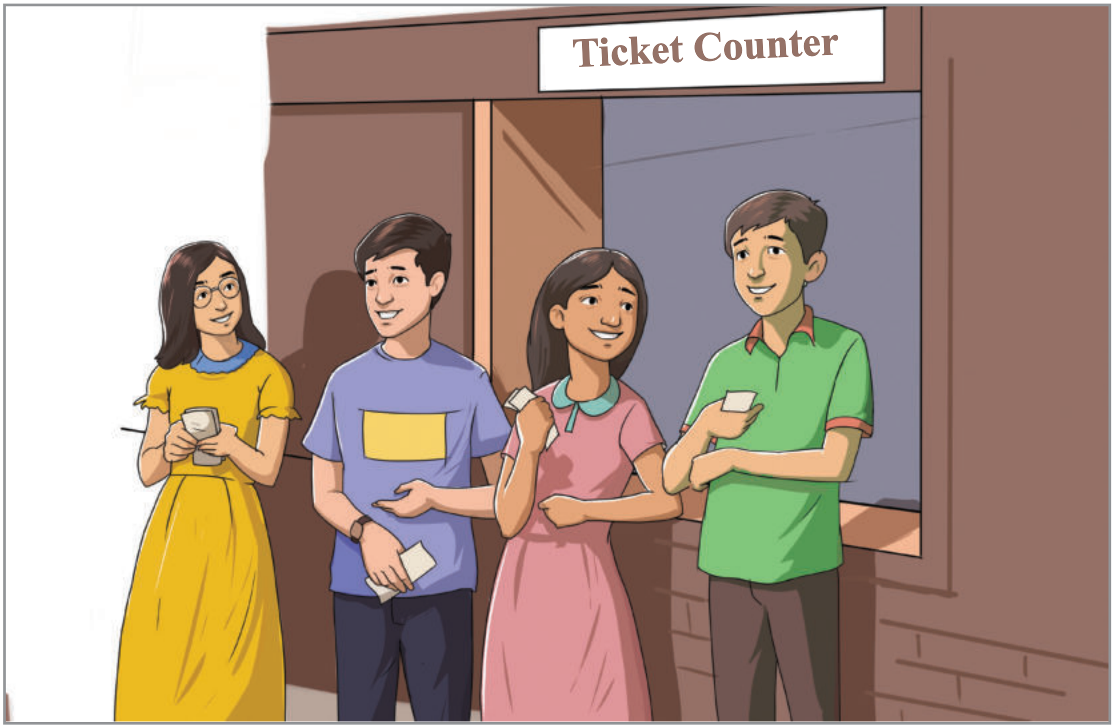
Now join the points using straight lines. Then, mark A(0, 7), which is the mid-value of the class interval which is right before the first class interval and mark H(21, 0), which is the mid-value of the class interval which is right after the last class interval. Now join A, B and G, H with straight lines. Then ABCDEFGH is our required polygon.

**Individual task:**

Draw Frequency Polygon without histogram using organized data of the group. Every member of the group will do this in their exercise book

## Analyzing data

Suppose, You four friends will go to visit Novo Theater.




There are various exhibition arrangements in Novo Theater and you have to buy ticket for those. Topu has tk. 80, Nitu has brought tk. 65, Himu has tk. 70 and you have tk. 75. Total amount is $\boxed{\phantom{000}}$ Reaching Novo Theater, you have found that the rate of one ticket for 5D film screening is tk. 40., ticket for virtual reality game is tk. 75. and ticket for planetarium exhibition is tk. 80. Which exhibition you four friends can see with which maximum amount of taka? How have you decided it? That is, You have determine the middle number of the sum total you four have altogether so that all of you can participate. This system is called average.

This is a simple example. But more complex question might come - Are the boys of Bangladesh more advanced in science education than the Girls? Or Who is the best cricket player of Bangladesh over the last ten years? Or Which grade students' attendance rate was the highest in your school? If we want to answer these questions the method of Statistics help us most is Central tendency. Using central tendency can find out the general characteristics of data.

## Central Tendency

Two tea stalls beside Nitu's school sell very good quality tea. Montu Mama owns one and the other's owner is Bindu Mashi.


On her way from home to school Nitu has noticed customers always crowd around both the tea stalls. But Nitu cannot decide which stall is earning more profit. So she collected all the data of last month. The data collected by Nitu are given in the box below.

Everyday profit of Montu mama's tea stall (in taka)

560, 615, 830, 670, 720, 920, 775, 920, 775, 720, 560, 615, 670, 920, 830, 775, 720, 775, 720, 775, 615, 670, 615, 720, 830, 720, 670,720, 830, 670

Everyday profit of Bindu Mashi's tea stall (in taka)

555, 730, 555, 780, 620, 825, 620, 730, 875, 620, 780, 660, 825, 660, 730, 780, 730, 730, 620, 730, 780, 660, 780, 825, 660, 825, 875, 660, 875, 730

**From the data above can Nitu tell which stall has earned more profit ? How shall she compare the data of these two stalls ?**

We can find some special number for every data which will represent the whole data. This special numbers are usually found around the centre of the data. That is, if raw data are arranged according to their value it accumulate around the mid value. Do the task given in the box .

## Individual task :

a) Organize the data collected from Montu mama's stall in the ascending order

b) Organize the data collected from Bindu Mashi's stall in the descending order

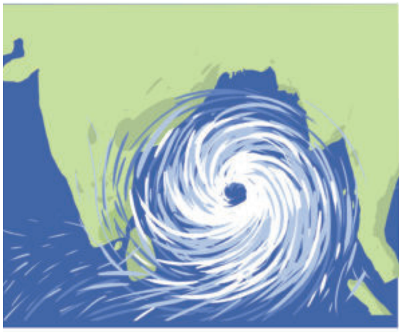
c) Organize the data collected from Bindu Mashi's stall in the regular order

**After arranging Do you see any number which are more frequent? If so , write it down.**

a) ________________

b) ________________

c) ________________



The speed of wind during Cyclone

The density of population of the capital of a country

Figure 10.5

**Can you guess anything looking at the two pictures above? Discuss with your partner. Write your idea briefly in your note book and show it to your teacher.**

If we present the raw data in a Frequency Distribution, there also we shall see in one class there will be highest Number of frequency. In fact the closest position of the Numbers is the central position. Therefore, we can say that the tendency of the data to accumulate towards the central value is called the Central tendency. Central value is a number and this number represent the data. Central tendency is measured with this number. The measures of Central Tendency are

1. Arithmetic average or Mean

2. Median 3. Mode।

What is the use of measuring central tendency ?

Suppose you have collected the monthly attendance of the students. If you can measure the central tendency of the data, you can easily say on which day the attendance was the highest or lowest or how many students remain present in most of the month. That is after collecting data regarding a particular incident if we find the measures of central tendency then we can explain the general characteristics of that data or incident quite easily

In this part of the experience you will measure the central tendency of the collected data in group after analyzing. Then you will take decision in order to determine the general characteristics of the collected data .Here you will get the idea about three measures Mean, Median and Mode.

- ☐ Where do we need to use Mean, Media and Mode?
- ☐ What kind of information do we get from Mean , median and Mode?
- ☐ What is the process of measuring Mean, median and Mode from raw data and the data in frequency table?

Things you have to consider with importance while doing the activities of this part are given in the box beside.

## Arithmetic Average or Mean)

Arithmetic Average or Mean In grade 6 you have learnt that if the total of the data is divided by the number of the data you get the average. Suppose t x 1he data of 30 days that Nitu collected from Montu Mama's stall $x_1 = 560$, $x_2 = 615$, $x_3 = 830$ … , $x_{30} = 670$

If the various $n$ values of the variable $x$ are $x_1, x_2, x_3, \dots \dots x_n$ and the average of the data is $\overline{x}$, then

$$\overline{x} = \frac{x_1 + x_2 + x_3 + \dots\dots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Here Σ is a Greek letter and it is called capital **sigma**, it means summation.

So total profit of last month of Montu Mama is –

$= x_1 + x_2 + x_3 \dots\dots\dots\dots + x_{30}$

= (560 + 615 + 830 + ….. + 670) tk

=21925 tk

∴ The average profit of the last month of Montu Mama

$= \frac{21925}{30}$ =730.83 tk (around )

(Calculator can be used if needed.)

This type of average is called Arithmetic mean or Mean

Well, isn't it a bit difficult to find the average if the numbers are very big ? What if we can find the average of big numbers with a little easier method. Let us see how much profit did Bindu Mashi earn tabling the data Nitu collected.

**Table 10.4**

| Amount of profit ($x_i$) | Number of Days ($f_i$) | $f_i x_i$ |
|---|---|---|
| 555 | 2 | 1110 |
| 620 | 4 | 2480 |
| 660 | 5 | 3300 |
| 730 | 7 | 5110 |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| | | |
|---|---|---|
| 780 | 5 | 3900 |
| 825 | 4 | 3300 |
| 875 | 3 | 2625 |
| Total | $n = 30$ | $\sum f_i x_i = 21825$ |

$\therefore$ The average profit of Bindu Mashi's tea stall = $\frac{1}{n}\sum f_i x_i = \frac{1}{30} \times 21825 = 727.50$ tk।

So we can see as per Nitu's information Montu mama's profit was more in the last month. We have shown how to determine Mean in two methods. Though this is a very easy way yet if the numbers are bigger this method can be time consuming. Moreover there is a scope of mistake also. Therefore, this is better determine Mean by tabling data in class distribution if the number/digit is bigger.

Let us try to find the mean by class distribution of the data collected by Nitu from Bindu Mashi's stall .

The smallest number from the data is 555 and the greatest number is 875. Now write what will be the scope of the data in the box beside :

Range = (________ − _________) + 1 = __________

Now if we take class interval 50 class number will be

= ( __________÷ 50) = 6.42 or __________

**Think and tell if the data is tabled how can we get the central value of every class?**

It is supposed that most of the frequencies of (every class) tabled data by class distribution centralize near the middle value. That is why the middle value of every class is considered its representative. So

Write a class and then write the higher limit and lower limit

Class_____________

Higher limit______Lower limit _________

$$\text{Class mid value} = \frac{\text{Lowest limit} + \text{highest limit}}{2}$$

Now let us make the Table 10.5

| Table 10.5 | | | | |
|---|---|---|---|---|
| Amount of profit (in tk) | Tally mark | Number of days ($f_i$) | Class mid value ($x_i$) | $f_ix_i$ |
| 550 – 600 | \|\| | 2 | 575 | 1150 |
| 601 – 650 | \|\|\|\| | 4 | 625 | 2500 |
| 651 – 700 | ~~\|\|\|\|~~ | 5 | 675 | 3375 |
| 701 – 750 | ~~\|\|\|\|~~ \|\| | 7 | 725 | 5075 |
| 751 – 800 | ~~\|\|\|\|~~ | 5 | 775 | 3875 |
| 801 – 850 | \|\|\|\| | 4 | 825 | 3300 |
| 851 – 900 | \|\|\| | 3 | 875 | 2625 |
| | | $n = 30$ | | $\sum f_i x_i = 21900$ |

$\therefore$ Arithmetic Mean of the last month $= \frac{1}{n}\sum f_i x_i = \frac{1}{30} \times 21900 = 730$ tk.

This new method of finding Arithmetic Average is called Direct Method

## Group Work

a) Find Arithmetic Average by the Direct method using the organized data

b) Does the team Rose think that the average height of the peers is perfect? Show reasons for your answer. If necessary you can use separate data – information of ideal height of boys and girls with their age .

c) Do the members of Dahlia think that compared to the average height of trees the growth of the trees is normal? Present your logic to other group.

d) Write it how calculator can help you from the table 10.4 and 10.5.

Notice the average profit from raw data of Bindu Mashi's stall was tk 772.50 and the average profit from processed data was tk 730, But the question is why have we got different value and which of them is accurate. Regarding raw data we know that exactly which data comes how many times .

For example, in Table 3 555 comes twice. But in processed data we only know that how many data are there in one class. Again in table 4 there are 2 data in the class (550 -600). But because of processing there is no scope to know that what are the values of these two data. That is why we have taken the mid value (575) of (550-600) as the representative of that class. It means we consider the value of 2 data of this class ( 550-600) is 575. Therefore this average is the average of raw data. But representative data of processed data and ------?----- data are not always the same. And if the data is changed is it possible to get right average?

We have learnt to do arithmetic average through direct method. Perhaps you are thinking if the average could be done in a bit easy method, it would have been better, Aren't you ? Perhaps you are thinking that if $x_i$ and $f_i$ is very big finding $f_i x_i$ and doing summation will be complex and time consuming. Even there is a chance of doing errors. So let us try to do Average using another method.

If you notice carefully you will see there is no change both table 10.4 and table 10.5. That is f i remains the same. Meantime we have learnt that Mean will be the central value of these $x_i$ So we can take one $x_i$ from the middle as Assumed Mean. Assumed Mean is identified with the sign (a). After tabling the raw data through class distribution the class interval (h) of every class remains the same. In that case finding class deviation $\left(u_i = \frac{x_i - a}{n}\right)$ and Mean can be determined in an easier and quicker way.

**This method of finding Arithmetic Mean is called Assumed mean Method or Short Method or Direct Method**

## Determining average using short or assumed mean method

| | |
|---|---|
| We can use the formula given beside to determine mean with short method | Arithmetic mean $(\bar{x}) = a + \frac{\sum f_i u_i}{n} \times h$ |

Because the difference between assumed mean of every class and every data is deviation. The average of this deviation or difference of average is assumed mean the difference of original mean.

If we add the average of this deviation or difference $\left(\frac{\sum f_i u_i}{n} \times h\right)$ we shall get the real value of mean.

Where. $\bar{x}$ = Arithmetic mean, $a$ = assumed mean, $n$ = total frequency, $h$ = class intervals and $\sum f_i u_i$ = sum of class interval multiplying with frequency of that intervals.

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Now let us find mean in short method using table 10.6

| Table 10.6 | | | | |
|---|---|---|---|---|
| Amount of profit In taka | Number of days $f_i$ | Midpoint of class $x_i$ | Class deviation $(u_i) = \frac{x_i - a}{h}$ | $f_i u_i$ |
| 550 – 600 | 2 | 575 | – 3 | – 6 |
| 601 – 650 | 4 | 625 | – 2 | – 8 |
| 651 – 700 | 5 | 675 | – 1 | –5 |
| 701 – 750 | 7 | 725 = (a) | 0 | 0 |
| 751 – 800 | 5 | 775 | 1 | 5 |
| 801 – 850 | 4 | 825 | 2 | 8 |
| 851 – 900 | 3 | 875 | 3 | 9 |
| | $n = 30$ | | | $\sum f_i u_i = 3$ |

Here, assumed mean is $a = 725$, total frequency $n = 30$, Deviation $h = 50$ and $\sum f_i u_i = 3$

$\therefore$ Arithmetic mean $\bar{x} = a + \frac{\sum f_i u_i}{n} \times h = 725 + \frac{3}{30} \times 50 = 725 + 5 = 730$

$\therefore$ Arithmetic Mean of profit is 730tk.

So we have seen that the value of mean that we have got both in the direct and short method from the unorganized data of profit from Bindu Mashi's stall is same. Therefore it can be said that to find mean in short method is more reliable if the class deviation is equal.

## Group work

- Find mean in short method using the organized data.
- Which method is easier and reliable explain and write it down in your exercise book

Write down the progress of your work so far

_______________

_______________

_______________

_______________

_______________

## Median

Suppose, monthly salary of four employees of an office are tk. 15000, 16000, 17000 and 18000. respectively. You have already learnt, how to do average. Calculate the average monthly salary of these four employees. Do this in the given box.

But monthly salary of the head of office is tk. 75000. Now, calculate monthly average salary of five employees including him. How much tk. did you get?

By this time you have come to know, Average is a number that represents data. If so, is the average you have found central value of all data? Or is it near the centre if arranged according to the value? If not, we have to find the reason. The reason is if one or two from the given data is much bigger or smaller than the other then the average value of the data does not remain near the central value. So here it can not give good results .

## But what is Median?

Median is another way to measure Central tendency. You got some ideas regarding Median in Grade Six. You have learnt that if the raw data data are arranged in right order Median divides the data into two equal parts.

After arranging the salary of five employees of office in ascending order identify the number that divides the value of data in equal halves.

Check if there is any change of mid value after arranging in descending order.

There were five employees including head of office. So you got 5 data. After arranging the data in right order you got one number in the middle and that is the Median. But if another employ would join with them and if his monthly salary would tk. 20000, then the data would be 6. Then how many data would you get in the middle and what would be the Median? Write your findings in the blank box below:

If you get two numbers in the middle position, then Median will be the average of those two numbers. So we can say if the number of raw data is $n$ and if is odd number, then Median will $\left(\frac{n+1}{2}\right)^{\text{th}}$ term. But if the n is even number then mode will be the average of numerical value of $\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2}+1\right)^{\text{th}}$ term

**Group Work:**

Find the Median from the collected data. From the Mean and Median you have got, which one is more effective? Explain with reasons why is it so.

Write how you will use Mean and median in your collected data

## Cumulative Frequency

Nitu and Shajal are collecting the information of weekly communication cost. They want to know how many friends have weekly communication cost below. tk 90 and how many of them have within tk. 70 to tk. 100.To do it, first they have listed the collected raw data. The list is shown below .

| Weekly Communication cost | 50 – 60 | 60 – 70 | 70 – 80 | 80 – 90 | 90 – 100 | 100 – 110 |
|---|---|---|---|---|---|---|
| Number of friends | 12 | 13 | 20 | 23 | 19 | 13 |

They have got 25 after adding the frequency of first class 12 and frequency of second class 13. This 25 will the cumulative frequency of second class. Since it has started with first class the cumulative frequency will remain 12. Again if you add third class frequency with the second class frequency we will get (20 + 25) = 45. This will be the third class frequency In this way, Nitu and Shajal have filled in some classes of the cumulative frequency Table. To fill in the rests is your responsibility, Can't you do?

**Table 10.7**

| Weekly comm. cost(in tk) | Number of Friends | Cumulative frequency | | Weekly comm. cost(in tk) | Cumulative frequency |
|---|---|---|---|---|---|
| 50 – 60 | 12 | 12 | | Less than 60 | 12 |
| 60 – 70 | 13 | 12 + 13 = 25 | | Less than 70 | 25 |
| 70 – 80 | 20 | 25 + 20 = 45 | → | Less than 80 | 45 |
| 80 – 90 | 23 | | | Less than 90 | |
| 90 – 100 | 19 | | | Less than 100 | |
| 100 – 110 | 13 | | | Less than 110 | |

Nitu and Shajal have known easily from table 6 how many friends' communication cost is less than tk. 90 and how many of them are from tk.70 to 100tk. Write down the number of friends in the following blank space of the box.

| | | |
|---|---|---|
| a) | | Weekly less than tk. 90 communication cost |
| b) | | Have cost from tk.70 to tk. 100 weekly |

### Group Work

Make a Cumulative frequency table from the class distribution frequency table of your collected data

## Why is Cumulative Frequency necessary to determine Median

All the 51 students of Nitu's class measured their height and made the table shown beside.

| Height (in cm) | 150 | 155 | 160 | 165 | 170 | 175 |
|---|---|---|---|---|---|---|
| Number of students | 4 | 6 | 12 | 16 | 8 | 5 |

The number of students in Nitu's class $n = 51$, which is an odd number. Therefore, the median of the heights of the students will be the value of $\left(\frac{n+1}{2}\right)^{th}$ term, that is, $\left(\frac{51+1}{2}\right)$th term or $26^{th}$ term. But in order to know the value and position of 26th term our students should learn cumulative number. Let us make the Cumulative frequency table first

**Table 10.8**

| Height (in cm) | 150 | 155 | 160 | 165 | 170 | 175 |
|---|---|---|---|---|---|---|
| Number of students | 4 | 6 | 12 | 16 | 8 | 5 |
| Cumulative Frequency | 4 | 10 | 22 | 38 | 46 | 51 |

From Cumulative frequency Table, we can see that from 23rd to 38th all terms have same value 165

$\therefore$ Value of 26th term = 165

$\therefore$ Median 165 cm.

We can see whether it is easy or not to take dicision presenting cumulative frequency in the table 10.8.

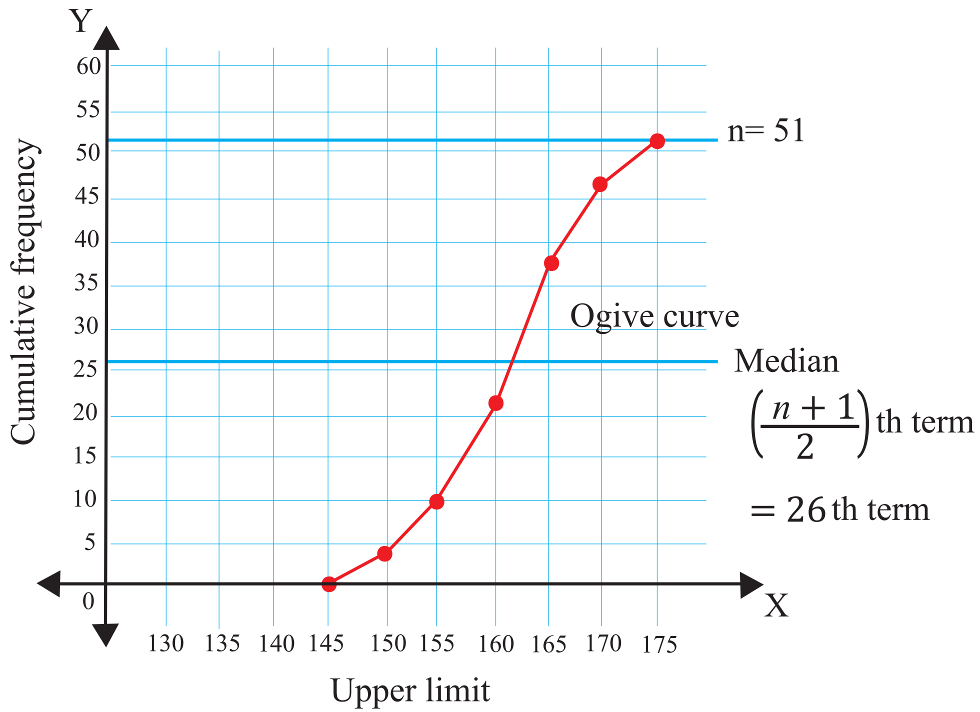



Figure 10.6

Let us present the table 7 of cumulative number in a graph paper, if we cansimplify the decision of determining Median value. In both the axes of a graph paper put the higher limit of every class in the vertical line or x axis and the cumulative frequency in the horizontal line or the y axis. If necessary different scale can be used for both the axes.

Now let us put the points (150, 4), (155, 10), (160, 22), (165, 38), (170, 46) and (175, 51) from the cumulative frequency of Table -7 in a the graph paper. Let us connect the points one after another without using a scale. As a result we get a curved line. This curved line is Cumulative frequency curve. Now we can more easily find the median by determining the value of 26th term = 165 $n$ = 51 for $\left(\frac{n+1}{2}\right)^{\text{th}}$ from this cumulative curve

## Individual task

Using the classified data find the Median from Cumulative Frequency Table and Cumulative Frequency Line graph or Cumulative Frequency Curve. Show your logic whether finding the Median has become easier because of drawing Cumulative Frequency Curve. Every member of the Group will do this task individually in their exercise book

## Finding median by using cumulative frequency curve

If the number of organized data is n, the value of $\left(\frac{n}{2}\right)^{\text{th}}$ term is median. And the $\left(\frac{n}{2}\right)^{\text{th}}$ will be in any of the class. The class in which $\left(\frac{n}{2}\right)^{\text{th}}$ term will be found will be median Class. But it will not be enough to know median class we need to find median also. We can use the following formula to find mode of organized data.

Where, L = the lower limit of the class of median, $n$ = total frequency, $F_c$ = cumulative frequency of the previous class of medianclass,
$f_m$= frequency of median class
$h$ = class interval

Median of organized data = $L + \left(\frac{n}{2} - F_c\right) \times \frac{h}{f_m}$

You know that the Median of any organized data is found in any class. But Can Median be less then the lowest limit? If not, then certainly the Median will be higher than the lower limit (L). The question is how higher? To find the answer to this question let us analyze the process of finding median by Nitu and Shajal.

Like your school garden there are various kinds of flower trees in Nitu's School garden. Along with the gardener Uncle all the students of Grade 8 take care of the garden by turn. Sometime they measure the height of the trees and length of the leaves to record whether the growth of the trees is normal. One day the following table is prepared by Nitu and her friend Punya after measuring the height of the trees.

| Table 10.9 | | | | | | |
|---|---|---|---|---|---|---|
| Height of Trees (in cm) | 100 – 110 | 110 – 120 | 120 – 130 | 130 – 140 | 140 – 150 | 150 – 160 |
| Number of Trees | 5 | 8 | 15 | 16 | 10 | 6 |

Nitu and her friend have decide that they will find the Median to know if the trees are growing up normally

| Table 10.10 | | | | | | |
|---|---|---|---|---|---|---|
| Height of Trees (in cm) | 100 – 110 | 110 – 120 | 120 – 130 | 130 – 140 | 140 – 150 | 150 – 160 |
| Number of Trees | 5 | 8 | 15 | 16 | 10 | 6 |
| Cumulative Frequency | 5 | 13 | 28 | 44 | 54 | 60 |

## They have done the following tasks to find median.

First they have drawn Cumulative Frequency Curve from Table 10.10

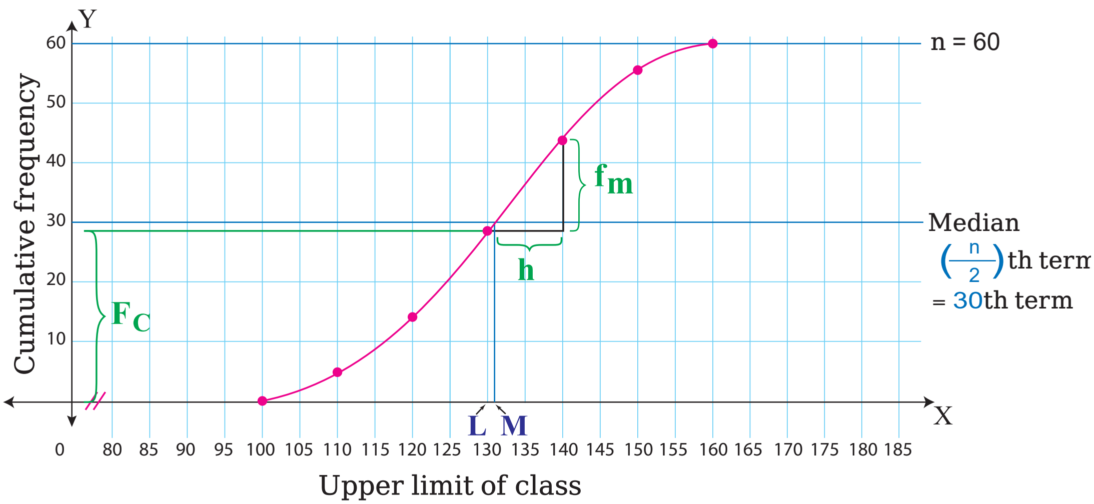



Figure 10.7

Meantime you have learnt that if the number of organized data is n, the value of $\left(\frac{n}{2}\right)$th term will be its median. And the $\left(\frac{n}{2}\right)$th will be in any of the class. The class in which $\left(\frac{n}{2}\right)$th term will be found will be median. Nitu and Punya collected the height of 60 trees as data. Therefore Total frequency $n = 60$, then, $\frac{n}{2} = \frac{60}{2}$ or 30th term will be the median. From the cumulative frequency curve we can see it is in the class (130 -140)

You can certainly remember the data are organized in a balanced manner. Here this will help us to find median. In the given figure/picture, the median that we have got from cumulative frequency curve is the position of M point. And L is the lower limit of the class of mode. $h$ = class interval.

$\therefore$ According to the table 10.10 L = 130, $F_c = 28$, $f_m = 16$, $h = 10$

Now observe the picture beside and you can write using the conception of triangle,

$$\frac{(M-L)}{h} = \frac{\left(\frac{n}{2} - F_c\right)}{f_m}$$

বা, $(M - L) = \left(\frac{n}{2} - F_c\right) \times \frac{h}{f_m}$

$\therefore M = L + \left(\frac{n}{2} - F_c\right) \times \frac{h}{f_m}$

সুতরাং, বাগানের গাছগুলোর উচ্চতার মধ্যক

$M = L + \left(\frac{n}{2} - F_c\right) \times \frac{h}{f_m}$

$= 130 + (30 - 28) \times \frac{10}{16}$

$= 130 + 2 \times \frac{5}{8} = 130 + 1.25$

$= 131.25$

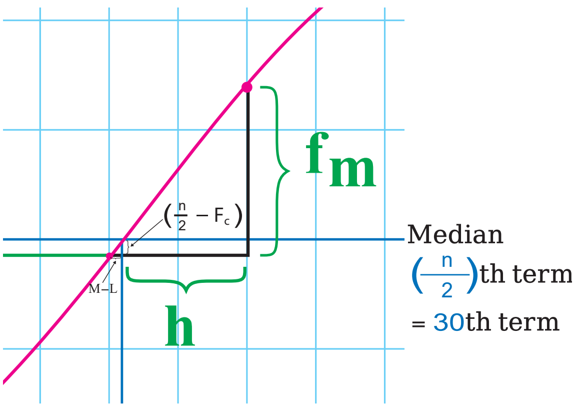



Figure 10.8

$\therefore$ Median = 131.25 cm (around) [You may use the calculator]

From the value of Median one can say that the length of half of the trees is less than 130.63cm and length of other half of the trees is more than 130.63 cm. We can get a general characteristic regarding the length of trees from the value of Median

**Group Work:**

Drawing a cumulative frequency Curve find the Median from the Classified frequency table of your collected data. Describe a general characteristics of data

## Mode

Nitu's school observes 14th March as Pi Day as yours/ school does. Initiatives have been taken to observe the day like every year. Nitu and her friends have decided they will arrange an interesting Quiz competition or magic show besides music, dance recitation, painting and acting. After the discussion, they divided 36 students of Grade six into 3 equal groups. Then they asked them to write the answers of 10 Puzzles to everyone of the first 12 members

The number of correct answers from 12 members of first group, are 6, 5, 4, 6, 7, 4, 6, 7, 3, 6, 8. After listing their information of correct answer they see that

| Number of correct answer $x_i$ | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| Number of student $f_i$ | 1 | 2 | 2 | 4 | 2 | 1 |

From this table we can see that the number of students who have given 6 correct answers is the highest. Can you say what will be the Mode of data in the table? In Grade six you have learned that the number among the data that appears the most is the Mode . So you say, what will be the Mode according to the information of the table?

Nitu and her friends have asked the 12 members of the second group to write another 10 interesting puzzles

The number of correct answers from 12 members of second group, are 5, 7, 4, 6, 7, 5, 4, 6, 4, 3, 6, 8

After listing their information of correct answer they see /find that

| Number of correct answer $x_i$ | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| Number of student $f_i$ | 1 | 3 | 2 | 3 | 2 | 1 |

From the table above 3 students have given 4 correct answers and 6 correct answers are given by 3 students. Here there are two Modes. These two Modes are 4 and 6. So mode may be one or more than one. At the end they gave to write each of the 3rd group a complex but intersting ten puzzel. The result is: each of twelve has given a correct answer

What is Mode ? Write answer with explanation

## Individual Task:

a) Collect the information of absence of the students of your class

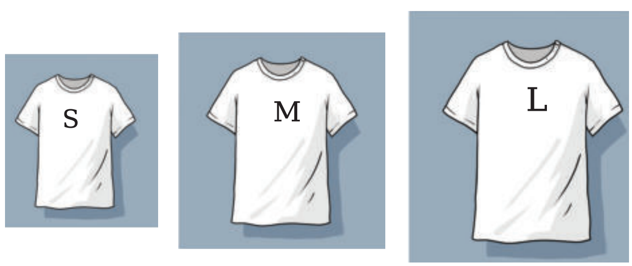
b) Identify the reasons of absence. Search and analyze the reasons. Review the identified causes.

c) Find the Mode of data if necessary.

d) Present a model or proposal to solve the problem.

## Why do we need Mode

You need to go to the market frequently to buy ready-made cloth and shoes. You have certainly noticed that small and medium sized



goods are mostly available, For example, for a particular brand shoe it is observed that number of available shoe size in 4 for the girls and 7 and 8 for the boys. To buy dresses for tall person is difficult, again size 10 shoes are not available in all shops. Can you guess the reason? Discuss with your classmate. Then write in two or three sentences in the blank box

## Finding Mode of organized data

In case of classified data it is not possible to find the Mode  from the data. You have to see which  class has most frequency. Because the Mode will be in that Mode. Here you can take help of Histogram.

## Finding Mode from Histogram

We are learning to draw Histogram. We can find Mode from Histogram. Remember, you draw Histogram of attendance of Grade Nine in table 10.2. Let us see how we can find Mode from that Histogram.

| Class of attendance | 8 – 10 | 10 – 12 | 12 – 14 | 14 – 16 | 16 – 18 | 18 – 20 |
|---|---|---|---|---|---|---|
| Number students | 6 | 16 | 24 | 34 | 12 | 8 |

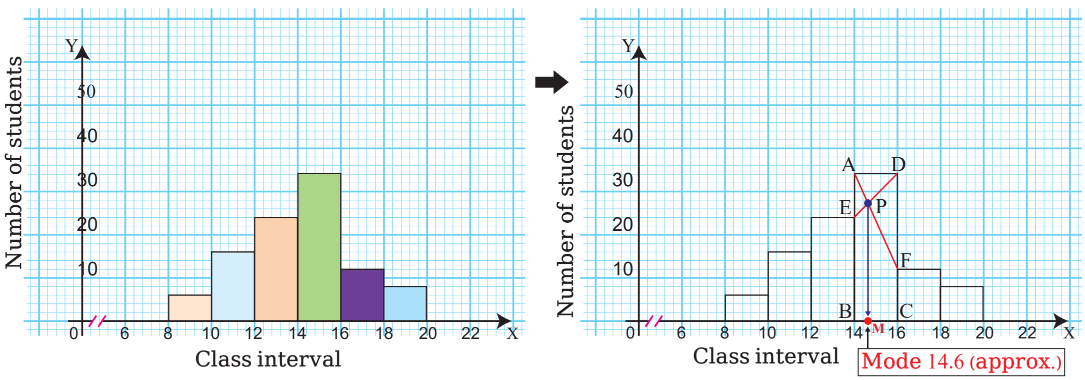



Figure 10.9

In the figure the histogram of modal class is ABCD (figure -10.9). Now let us connect A, F and E, D. AF and ED has intersected one another in the point P. Let us draw PM vertical line on the *x* - axis from P point. Which has intersected the *x* axis in the M point. The  *x* coordinate of M point is (14.6 ,0). This X coordinate of M will be the Mode of collected data about the attendance of grade nine students. Hence the Mode is 14.6 (around)

We can determine mode of organized data using the formula.

To find mode from the graph paper there might be possibility of doing mistake to determine $x$ coordinate. If we apply direct geometrical method, we shall get the value more easily.

While determining median we applied the concept of triangle to solve the problem. We got an easy solution. Let us form/ establish a formula in the same way.

To find Mode of organized data the formula we have got, Mode $= L + \frac{f_1}{f_1+f_2} \times h$

Where, L = the lower limit of modal class, $n$ = total frequency, $f_1$ = the difference of frequency of modal class and the preceding class frequency , $f_2$ = the difference between the frequency of modal class and succeeding class frequency and $h$ = class interval

Like you Nitu collected the information of weekly income about 40 families of her village. Here is the organized list of collected data :

| Weekly income | 4000–5000 | 5000–6000 | 6000–7000 | 7000–8000 | 8000–9000 | 9000–10000 |
|---|---|---|---|---|---|---|
| Number of family | 5 | 8 | 12 | 10 | 3 | 2 |

Let us find mode using Nitu's list applying the formula :

You know that to find mode of data you have to determine the modal class, According to the list the highest income of 12 families out of 40 is ( 6000 -70000 )

Therefore the modal class in the list is (6000 – 7000)

$\therefore$ mode of classified data $= \mathbf{L} + \frac{f_1}{f_1+f_2} \times \boldsymbol{h}$

$= 6000 + \frac{4}{4+2} \times 1000 = 6000 + \frac{4}{6} \times 1000$

$= 6{,}666.67$ tk

$L = 6000$
$f_1 = (12 - 8) = 4$
$f_2 = (12 - 10) = 2$
এবং $h = 1000$

But class of mode of organized data may not remain on the middle of those classes. Some cases this may be remain in the first class or in the last class. In that case, what will be happen?

Now observe another incident. Nitu along with the list of weekly income of every family, has collected the information of age of every villagers in years and prepared the given list.

| Age in year | 1 – 10 | 11 – 20 | 21 – 30 | 31 – 40 | 41 – 50 | 51 – 60 | 61 – 70 |
|---|---|---|---|---|---|---|---|
| Number of population | 82 | 27 | 25 | 52 | 50 | 32 | 12 |

From the list we can see that among the forty families number of children of the age range 1-10 is the highest. Therefore, modal class in the list will be L = 1, $f_1 = (82 - 0) = 82$, $f_2\ (82 - 27) = 55$ and $h = 10$

$\therefore$ Mode of organized data $= L + \frac{f_1}{f_1+f_2} \times h = 1 + \frac{82}{82+55} = 1 + \frac{82}{137} \times 10$

$= 6.99$ years (approx.)

Again suppose the following list is prepared after taking the weight of some students of Grade Ten:

| Weight in kg | 35 – 40 | 40 – 45 | 45 – 50 | 50 – 55 | 55 – 60 |
|---|---|---|---|---|---|
| Number of students | 2 | 3 | 10 | 18 | 32 |

Here frequency is maximum 32 times in the interval (55-60). So mode class is (55-60)

So $L = 55, f_1 = (32 - 18) = 14, f_2 = (32 - 0) = 32$ and $h = 5$

$\therefore$ Mode of organized data $= L + \frac{f_1}{f_1+f_2} \times h = 55 + \frac{14}{14+32} \times 5 = 55 + \frac{14}{46} \times 5$

$= 56.52$ Kg. (approx.)

## Group Work

a) Find Mode from Histogram of classified data.

b) Verify/check finding Mode using formula.

c) What decision can be taken regarding collected data from the value of mode.

In this experience you have collected and analyzed data through group activity. You have explained some general characteristics from the results of the analysis. Statistics is basically a science of taking decision through collecting and analyzing data. The steps you followed in collecting data, presenting data and measuring Central tendency are used in any decision taking in our everyday life. Every measurement of Central Tendency namely Mean, Median and Mode contributes in identifying general characteristics of any incident and taking decision about that incident. Therefore, it is necessary to be skilled in measuring Central tendency after analyzing raw or processed data using different method described above. Simultaneously, it is very essential to learn how to use Histogram, Frequency Polygon, Cumulative Frequency Curve to present data.

## Exercise

1. The height (in cm) of some students of Grade VIII are given in the box beside. Solve the following problem.

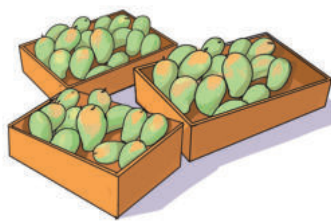
90, 140, 97, 125, 97, 134, 97, 97, 110, 125, 110, 134, 110, 125,110, 140, 125, 134, 125, 125, 134, 110, 125, 97, 125, 110, 125, 97,134, 125, 110, 134, 125,134, 90,140, 148, 148, 110, 125

a) Arrange the data in ascending order.

b) Arrange the data in descending order.

c) Find the average height of the studen.t

2. Mr. Mizan is a mango seller. He has bought 50 boxes of mango.In every box the number of mangoes is not equal. But we need to know how many mangoes are there in each box. Find out how many mangoes are there in the 50 boxes from the given Table.



| Number of mango | 51 – 53 | 54 – 56 | 57 – 59 | 60 – 62 | 63 – 65 |
|---|---|---|---|---|---|
| Number of Box | 6 | 14 | 16 | 9 | 5 |

3. Look at the graph beside

a) Name the graph

b) What kind of data are these

c) What is its Median?

d)Make a table from the graph

e) Find Mean Median and Mode from the table

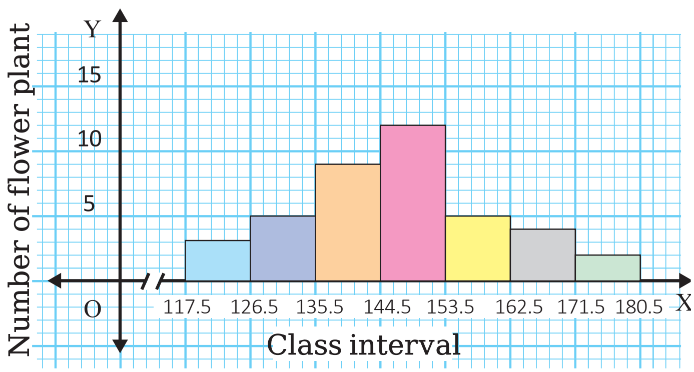



Figure 10.10

4.

| Class | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 |
|---|---|---|---|---|---|
| Frequency | 7 | 11 | p | 9 | 13 |

If the arithmetic average is 54 what will be the value of p (in direct method). After that verify the value of p with the help of short method.

5. In the Table below the wages (in taka) of the worker of a garment factory is given. If the mode is 525 determine the value of $x$ and $y$. The total number of workers in the factory is 120.

| Daily wages (in taka) | Number of worker |
|---|---|
| 300 – 400 | 12 |
| 400 – 500 | 20 |
| 500 – 600 | $x$ |
| 600 – 700 | 30 |
| 700 – 800 | $y$ |
| 800 – 900 | 5 |
| 900 – 1000 | 4 |

6. Find out the number of 100 patients of a health care centre from the list of class interval and cumulative frequency of the Patients' age.

| Age (in year) | 0 – 10 | 11 – 20 | 21 – 30 | 31 – 40 | 41 – 50 | 51 – 60 | 61 – 70 |
|---|---|---|---|---|---|---|---|
| Number of patient | | | | | | | |
| Cumulative Frequency | 5 | 9 | 24 | 41 | 68 | 85 | 100 |

7. The table of daily Profit ( in taka) of 100 shops at Nagri bazar

| Profit of each shop (in tk. ) | 300 – 350 | 350 – 400 | 400 – 450 | 450 – 500 | 500 – 550 | 550 – 600 |
|---|---|---|---|---|---|---|
| Number of shop | 10 | 16 | 28 | 22 | 18 | 6 |

a) Make a table of cumulative frequency based on the given information.

b) How many shops have earned profit less than tk. 500?

8. The list is made after processing raw data of the age (in year) of all the family members of grade eight students.

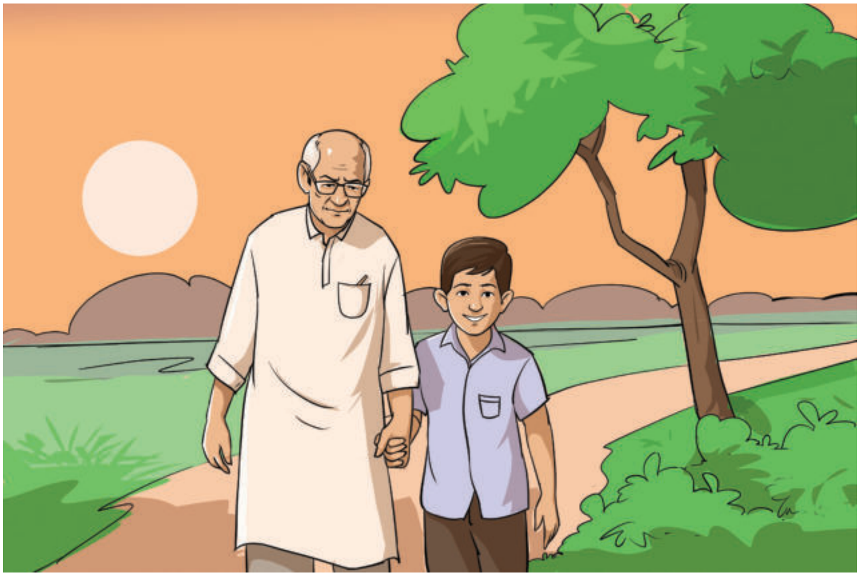
| Age (year ) | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 |
|---|---|---|---|---|---|---|---|
| Frequency | 30 | 60 | 82 | 94 | 66 | 48 | 20 |

a) Draw a histogram of the data.

b) Draw frequency polygon from the histogram.

c) Draw frequency polygon without histogram.

9. Everyday Shajal goes to the nearest park with his grandfather for morning walk. He has planned to collect information according to their age.



The table of the collected data is :

| Age (year) | 41 – 45 | 46 – 50 | 51 – 55 | 56 – 60 | 61 – 65 |
|---|---|---|---|---|---|
| Frequency | 12 | 15 | 25 | 18 | 10 |

| | |
|---|---|
| a) Find mean of the data both in direct and indirect method.<br>b) Find mode of the data.<br>c) Draw histogram using the list of Shajal's information. | d) Find mode.<br>e) Draw frequency Polygon of the data.<br>f) Draw cumulative frequency curve. |

10. Suppose sometime there is load shedding / power cut in your locality. Make a plan to solve the problem. You can do the following task according to the plan.

    a) Collect the bills of electricity of neighbouring family.

    b) To know how much electricity the family use, Table the data .

    c) Present your opinion and proposal for duties as per the demand of electricity.

11.

    i) Collect the information of the age (in year) of 25 members including family members and near relatives and record it. (take help from your guardian if needed)

    ii) Graph of collected information of 30 members of your friend's family including the near relatives .

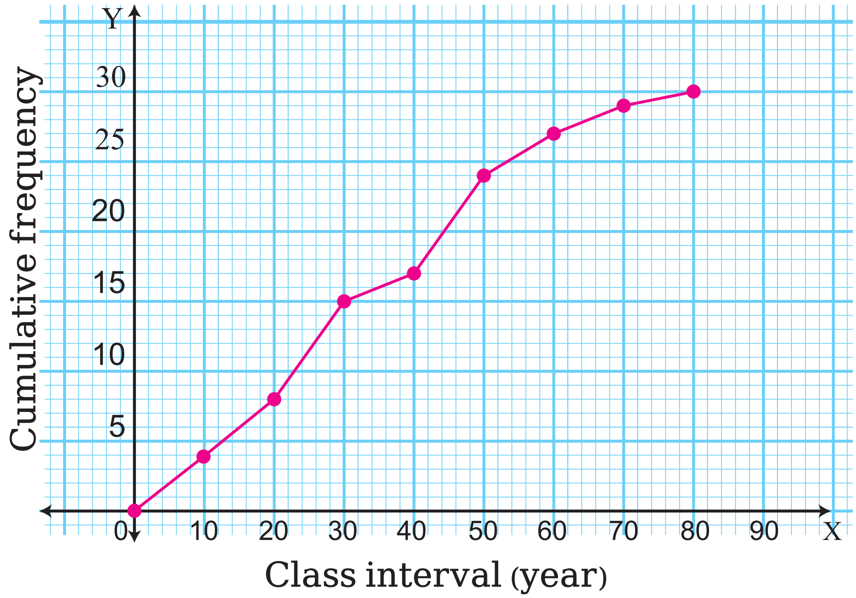



Figure 10.11

Using the data of i)

a) Make a table of frequency.

b) Draw a histogram and find frequency of polygon and Mode.

c) Find arithmetic average.

d) Find median and mode.

e) Make a table using the figure of ii).

f) Write down the comparative difference between your family members and your friend's family members. Explain whether the numbers of members age, and class interval influence the average.

g) Which is easier for presenting data? Graph or table? Give reasons for your answer.

12. Arrange the task of your group from data collection to information analysis to decision making . In every step write a brief description of your task and present it .

Here the tasks are in a random order. Leave the step which you did not have to follow.

Organizing data ➡ collecting data ➡ classifying data ➡ verifying reliability of source ➡ determining scope ➡ selecting source ➡ finding class interval ➡ finding median and mode ➡ finding central tendency ➡ finding arithmetic mean ➡ taking decision about data from the value of central tendency ➡ finding cumulative frequency ➡ explaining the value of median and mode ➡ finding mode from histogram

সৌরবিদ্যুৎ চালিত সেচ পাম্প

প্রধানমন্ত্রী 'শেখ হাসিনার উদ্যোগ ঘরে ঘরে বিদ্যুৎ' এই শ্লোগানকে সামনে নিয়ে প্রচলিত পদ্ধতিতে বিদ্যুৎ উৎপাদনের পাশাপাশি নবায়নযোগ্য জ্বালানি যেমন, সৌরবিদ্যুৎ, উইন্ডমিল ও বায়োগ্যাস থেকেও বিদ্যুৎ উৎপাদিত হচ্ছে। সূর্য থেকে বিকিরণ হওয়া তাপশক্তিকে রাসায়নিক বিক্রিয়ার মাধ্যমে কাজে লাগিয়ে যে বিদ্যুৎ উৎপন্ন করা হয় তাই হলো সৌরবিদ্যুৎ। বাংলাদেশে অফ-গ্রিড এলাকায় (চর, হাওড় ও দুর্গম পাহাড়ি এলাকা) সৌরবিদ্যুৎ মানুষের জীবনযাত্রার মানে পরিবর্তন এনেছে। জাতীয় প্রবৃদ্ধি অর্জন, দারিদ্র্য বিমোচন এবং দেশের আর্থ-সামাজিক উন্নয়নের অন্যতম চালিকাশক্তি বিদ্যুৎ। দেশের বিদ্যুৎ খাতে অভূতপূর্ব উন্নয়নের ফলে অবকাঠামো, কৃষি ও শিল্প খাতে ইতিবাচক প্রভাব পড়েছে এবং নতুন কর্মসংস্থান সৃষ্টি হয়েছে। সৌরবিদ্যুৎ পরিবেশ-বান্ধব হওয়ায় বেসরকারি পর্যায়ে ভবনের ছাদে সৌরবিদ্যুৎ উৎপাদন জনপ্রিয় করার জন্য 'নেট মিটারিং গাইডলাইন' প্রণয়ন করা এবং বিদ্যুৎবিহীন এলাকার শিক্ষাপ্রতিষ্ঠানে অগ্রাধিকার ভিত্তিতে সোলার প্যানেল স্থাপন করা হচ্ছে।

Academic Year 2024

Class Eight

# Mathematics

সমৃদ্ধ বাংলাদেশ গড়ে তোলার জন্য যোগ্যতা অর্জন করো

– মাননীয় প্রধানমন্ত্রী শেখ হাসিনা

## বিদ্যা পরম ধন

তথ্য, সেবা ও সামাজিক সমস্যা প্রতিকারের জন্য '৩৩৩' কলসেন্টারে ফোন করুন

নারী ও শিশু নির্যাতনের ঘটনা ঘটলে প্রতিকার ও প্রতিরোধের জন্য ন্যাশনাল হেল্পলাইন সেন্টার ১০৯ নম্বর-এ (টোল ফ্রি, ২৪ ঘণ্টা সার্ভিস) ফোন করুন

Ministry of Education